6.4

Solving Trigonometric Equations Using Identities

Focus on...

- solving trigonometric equations algebraically using known identities
- determining exact solutions for trigonometric equations where possible
- determining the general solution for trigonometric equations
- identifying and correcting errors in a solution for a trigonometric equation

Examples:

Solve each equation algebraically over the domain indicated.

(a) $\sin 2x - \cos x = 0$ $\qquad 0 \le x \le 2\pi$

- no common factors

$\sin 2A = 2\sin A\cos A$

$2\sin x\cos x - \cos x = 0$

- common factor $\cos x$

$\cos x\left[2\sin x - 1\right] = 0$

- set each factor equal to zero and solve for x.

$\cos x = 0$

$x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi n, n \in I$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \begin{cases} \frac{\pi}{6} \\ \frac{5\pi}{6} \end{cases} + 2\pi n, n \in I$

interval:

$x = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}\right.$

(b) $2\cos x + 1 - \sin^2 x = 3$ $\quad 0 \le x \le 2\pi$ $\quad \cos\theta = \frac{A}{H}$

- can't do anything as it is written
- use identity $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$2\cos x + 1 - (1 - \cos^2 x) = 3$

$2\cos x + \cos^2 x = 3$

- Quadratic to factor

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x - 1)(\cos x + 3) = 0$

$\cos x = 1 \qquad \cos x = -3$

$\cos x = -3 \qquad -1 \le \cos x \le 1$

NO SOL'N

$\cos x = 1$

$x = \{0 + 2\pi k, k \in I$

Interval

$x = \{0, 2\pi$

(c) $\sin^2 x = \frac{1}{2}\tan x \cos x \qquad 0° \le x \le 360°$

• use identity $\tan x = \frac{\sin x}{\cos x}$

we have introduced a fraction
we **MUST** look for NPV

$\cos x \neq 0$

$x \neq \begin{cases} 90° \\ 270° \end{cases} + 360°n \quad n \in I$

$\sin^2 x = \frac{1}{2}\left(\frac{\sin x}{\cancel{\cos x}}\right)\cancel{\cos x}$

$\sin^2 x = \frac{1}{2}\sin x$

$\sin^2 x - \frac{1}{2}\sin x = 0$

• factor

$\sin x\left(\sin x - \frac{1}{2}\right) = 0$

$\sin x = 0 \qquad \sin x = \frac{1}{2}$

$x = \begin{cases} 0° \\ 180° \end{cases} + 360°n \qquad x = \begin{cases} 30° \\ 150° \end{cases} + 360°n$

interval:

$x = \begin{cases} 0°, 180°, 360° \\ 30°, 150° \end{cases}$

★check these with NPV (we are good)

(d) $\cos 2x = \cos x$ all solutions (radians)

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

let $m = \cos x$

$2m^2 - m - 1 = 0$

$(2m+1)(m-1) = 0$

$m = -\frac{1}{2}$ $m = 1$

$\cos x = -\frac{1}{2}$

$$\boxed{x = \begin{cases} \frac{2\pi}{3} \\ \frac{4\pi}{3} \end{cases} + 2\pi k, k \in I}$$

$\cos x = 1$

$$\boxed{x = \{ 0 + 2\pi k, k \in I}$$

(e) $3\cos x + 2 = 5\sec x$ all solutions (radians)

• use identity $\sec\theta = \frac{1}{\cos\theta}$

NPV: $\cos x \neq 0$

$x \neq \left\{ \begin{array}{l} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{array} \right. + 2\pi k, k \in I$

$3\cos x + 2 = \frac{5}{\cos x}$

• mult. each term by cos x

$3\cos^2 x + 2\cos x = 5$

$3\cos^2 x + 2\cos x - 5 = 0$

$(3\cos x + 5)(\cos x - 1)$

$3\cos x + 5 = 0$ $\quad \cos x - 1 = 0$

$\cos x = -\frac{5}{3}$

NO SOL'N

$\cos x = 1$

$x = \{0 + 2\pi k, k \in I$

→ check with NPV... we're good :)

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Example: Solve the following for the indicated interval in radians

A) $\cos x \cos\left(\frac{\pi}{5}\right) - \sin x \sin\left(\frac{\pi}{5}\right) = \frac{\sqrt{3}}{2}$ (all solutions)

compound angle: let $x = A$

$\frac{\pi}{5} = B$

$\cos A \cos B - \sin A \sin B = \frac{\sqrt{3}}{2}$

$\cos(A+B) = \frac{\sqrt{3}}{2}$

$\cos\left(x + \frac{\pi}{5}\right) = \frac{\sqrt{3}}{2}$

let $m = x + \frac{\pi}{5}$

$\cos m = \frac{\sqrt{3}}{2}$

$m = \begin{cases} \frac{\pi}{6} \\ \frac{11\pi}{6} \end{cases} + 2\pi k, k \in I$

$x + \frac{\pi}{5} = \begin{cases} \frac{\pi}{6} \\ \frac{11\pi}{6} \end{cases} + 2\pi k, k \in I$

$$x+\frac{\pi}{5}=\begin{cases}\frac{\pi}{6} \\ \frac{11\pi}{6}\end{cases}+2\pi k,\ k\in I$$

• get a common denominator

$$x+\frac{6\pi}{30}=\begin{cases}\frac{5\pi}{30} \\ \frac{55\pi}{30}\end{cases}+2\pi k,\ k\in I$$

$$\boxed{x=\begin{cases}\frac{-\pi}{30} \\ \frac{49\pi}{30}\end{cases}+2\pi k,\ k\in I}$$

B) $4\sin x\cos x=\sqrt{3}$ $\quad -2\pi\le x\le 2\pi$ $\quad \frac{12\pi}{6}$ $\frac{6\pi}{3}$

$\frac{2(2\sin x\cos x)}{2}=\frac{\sqrt{3}}{2}$

$2\sin x\cos x=\frac{\sqrt{3}}{2}$

$\sin 2x=\frac{\sqrt{3}}{2}$

$\left(\frac{1}{2}\right)2x=\begin{cases}\frac{\pi}{3}\left(\frac{1}{2}\right)\\ \frac{2\pi}{3}\left(\frac{1}{2}\right)\end{cases}+2\pi k\left(\frac{1}{2}\right), k\in I$

$\frac{6\pi}{6}, \frac{3\pi}{3}$

$x=\begin{cases}\frac{\pi}{6}\\ \frac{\pi}{3}\end{cases}+\pi k, k\in I$

Interval:

$x=\begin{cases}\frac{\pi}{6}, \frac{7\pi}{6}, \frac{-5\pi}{6}, \frac{-11\pi}{6}\\ \frac{\pi}{3}, \frac{4\pi}{3}, \frac{-2\pi}{3}, \frac{-5\pi}{3}\end{cases}$

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