Proving Trig Identities – Day 3

Examples:

1. Prove: $1 - \tan x \tan y = \dfrac{\cos(x+y)}{\cos x \cos y}$

$$\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$\frac{\cancel{\cos x}\,\cancel{\cos y}}{\cancel{\cos x}\,\cancel{\cos y}} - \left(\frac{\sin x}{\cos x}\right)\left(\frac{\sin y}{\cos y}\right)$$

$$1 - \tan x \tan y$$

2. Prove: $\dfrac{\sin 3x - \sin x}{\cos 3x + \cos x} = \tan x$

LHS=

$$\frac{\sin(2x+x) - \sin x}{\cos(2x+x) + \cos x}$$

$$\frac{(\sin 2x\cos x + \cos 2x\sin x) - \sin x}{(\cos 2x\cos x - \sin 2x\sin x) + \cos x}$$

$$\frac{(2\sin x\cos x)\cos x + \cos 2x\sin x - \sin x}{\cos 2x\cos x - (2\sin x\cos x)\sin x + \cos x}$$

* $\cos 2A = \cos^2 A - \sin^2 A$

* $= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

*

$$3x(y-1)$$

$$3xy-3x$$

$$3x(2xy)$$

$$6x^2y$$

$$\frac{(2\sin x\cos x)\cos x+\sin x(\cos 2x)-\sin x}{(\cos 2x)\cos x-(2\sin x\cos x)\sin x+\cos x}$$

$$\frac{2\sin x\cos^2 x+\sin x(\cos^2 x-\sin^2 x)-\sin x}{(\cos^2 x-\sin^2 x)\cos x-2\sin^2 x\cos x+\cos x}$$

using $\cos 2x=\cos^2 x-\sin^2 x$

$$\frac{\sin x\left[2\cos^2 x+\cos^2 x-\sin^2 x-1\right]}{\cos x\left[\cos^2 x-\sin^2 x-2\sin^2 x+1\right]}$$

$$\tan x\left[\frac{3\cos^2 x-\sin^2 x-1}{\cos^2 x-3\sin^2 x+1}\right]$$

$$\tan x\left[\frac{3\cos^2 x-\sin^2 x-1}{\cos^2 x-3\sin^2 x+1}\right]$$

$$\tan x\left[\frac{3\cos^2 x-[1-\cos^2 x]-1}{\cos^2 x-3[1-\cos^2 x]+1}\right]$$

$$\tan x\left(\frac{4\cos^2 x-2}{\cos^2 x-3+3\cos^2 x+1}\right)$$

$$\tan x\left(\frac{4\cos^2 x-2}{4\cos^2 x-2}\right)$$

$$\tan x$$

$$\frac{(2\sin x\cos x)\cos x+\sin x(\cos 2x)-\sin x}{(\cos 2x)\cos x-(2\sin x\cos x)\sin x+\cos x}$$

$$\frac{2\sin x\cos^2 x+\sin x(1-2\sin^2 x)-\sin x}{(1-2\sin^2 x)\cos x-2\sin^2 x\cos x+\cos x}$$

using $\cos 2x = 1-2\sin^2 x$

$$\frac{\sin x\left[2\cos^2 x+1-2\sin^2 x-1\right]}{\cos x\left[1-2\sin^2 x-2\sin^2 x+1\right]}$$

$$\tan x\left[\frac{2\cos^2 x-2\sin^2 x}{2-4\sin^2 x}\right]$$

$$\tan x\left[\frac{2\cos^2 x - 2\sin^2 x}{2 - 4\sin^2 x}\right]$$

$$\tan x\left(\frac{2(\cos^2 x - \sin^2 x)}{2(1 - 2\sin^2 x)}\right)$$

$$\tan x\left(\frac{\cos 2x}{\cos 2x}\right)$$

$$\tan x$$

$$\frac{(2\sin x\cos x)\cos x + \sin x(\cos 2x) - \sin x}{(\cos 2x)\cos x - (2\sin x\cos x)\sin x + \cos x}$$

using $\cos 2x = 2\cos^2 x - 1$

$$\frac{2\sin x\cos^2 x + \sin x\,(2\cos^2 x - 1) - \sin x}{(2\cos^2 x - 1)\cos x - 2\sin^2 x\cos x + \cos x}$$

$$\frac{\sin x\left[2\cos^2 x + 2\cos^2 x - 1 - 1\right]}{\cos x\left[\,2\cos^2 x - 1 - 2\sin^2 x + 1\,\right]}$$

$$\tan x\left[\frac{4\cos^2 x - 2}{2\cos^2 x - 2\sin^2 x}\right]$$

$$\tan x\left[\frac{4\cos^2 x-2}{2\cos^2 x-2\sin^2 x}\right]$$

$$\tan x\left[\frac{2(2\cos^2 x-1)}{2(\cos^2 x-\sin^2 x)}\right]$$

$\sin^2\theta = 1-\cos^2\theta$

$$\tan x\left(\frac{\cos 2x}{\cos 2x}\right)$$

$$\tan x$$