SCO T06 Students will be expected to prove trigonometric identities, using

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities (restricted to sine, cosine, and tangent)
- double-angle identities (restricted to sine, cosine, and tangent)

Ch 6.3 Proving Identities DAY 2

Prove that $\frac{\sin 2x}{\cos 2x + 1} = \tan x$ is an identity for all permissible values of x.

$$\frac{\sin 2x}{\cos 2x+1} \quad = \quad \tan x$$

All 3 cos2A will work, there will be one that is more efficient

$$\frac{2\sin x\cos x}{(2\cos^2 x-1)+1}$$

$$\frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos x\cdot\cancel{\cos x}}$$

$$\frac{\sin x}{\cos x}$$

$$\tan x \qquad = \tan x$$

Prove: $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$$\frac{2\tan x}{\sec^2 x}$$

Pythag. Ident

$\tan^2\theta + 1 = \sec^2\theta$

$$2\tan x\left(\frac{1}{\sec^2 x}\right)$$

$$2\left(\frac{\sin x}{\cos x}\right)\left(\cos^2 x\right)$$

$$\frac{2\sin x \cdot \cos x \cdot \cancel{\cos^2 x}}{\cancel{\cos x}}$$

$$2\sin x\cos x$$

$$\sin 2x$$

$= \sin 2x$

Prove that $\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1} = \dfrac{\sin^2 x\cos x + \cos^3 x}{2\sin x + 1}$ is an identity for all permissible values of x.

common factor

$\dfrac{\cos x(\sin^2 x + \cos^2 x)}{2\sin x + 1}$

Pythag Ident

$\dfrac{\cos x}{2\sin x + 1}$

mult. by a fancy 1

conjugate

$\dfrac{\cos x}{2\sin x + 1}\left(\dfrac{2\sin x - 1}{2\sin x - 1}\right)$

$\sin 2x$

$\dfrac{2\sin x\cos x - \cos x}{4\sin^2 x - 1}$

$\dfrac{\sin 2x - \cos x}{4\sin^2 x - 1}$

Prove that $\frac{\sin 2x - \cos x}{4\sin^2 x - 1} = \frac{\sin^2 x \cos x + \cos^3 x}{2\sin x + 1}$ is an identity for all permissible values of x.

$$\frac{2\sin x\cos x - \cos x}{(2\sin x+1)(2\sin x-1)}$$

$$\frac{\cos x(\cancel{2\sin x-1})}{(2\sin x+1)(\cancel{2\sin x-1})}$$

$$\frac{\cos x}{2\sin x+1}$$

$$\frac{\cos x(\sin^2 x + \cos^2 x)}{2\sin x+1}$$

$$\frac{\sin^2 x\cos x + \cos^3 x}{2\sin x+1}$$

think about fancy 1

$1 = \sin^2 x + \cos^2 x$

$$\sec 2a + \tan 2a = \frac{\cos a + \sin a}{\cos a - \sin a}$$

$$\frac{1}{\cos 2a} + \frac{\sin 2a}{\cos 2a}$$

$$\frac{1 + \sin 2a}{\cos 2a}$$

$$\frac{1 + 2\sin a \cos a}{\cos^2 a - \sin^2 a}$$

$$\frac{1 + 2\sin a \cos a}{(\cos a - \sin a)(\cos a + \sin a)}$$

$$\frac{1+2\sin a\cos a}{(\cos a-\sin a)(\cos a+\sin a)}$$

$$\frac{\cos^2 a+\sin^2 a+2\sin a\cos a}{(\cos a-\sin a)(\cos a+\sin a)}$$

$$\frac{(\cos a+\sin a)\cancel{(\cos a+\sin a)}}{(\cos a-\sin a)\cancel{(\cos a+\sin a)}}$$

$$\frac{\cos a+\sin a}{\cos a-\sin a}$$

fancy $1 = \sin^2\theta + \cos^2\theta$

sometimes it is helpful to rearrange terms

let $x = \cos a$ $y = \sin a$

x^2+y^2+2xy

$x^2+2xy+y^2$ ← perfect square

$(x+y)(x+y)$