6.3

Proving Identities

Focus on...

- proving trigonometric identities algebraically
- understanding the difference between verifying and proving an identity
- showing that verifying that the two sides of a potential identity are equal for a given value is insufficient to prove the identity

SCO T06 Students will be expected to prove trigonometric identities, using

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities (restricted to sine, cosine, and tangent)
- double-angle identities (restricted to sine, cosine, and tangent)

Examples: Prove the following identities:

1. $\sin x \cos x \cot x = \cos^2 x$

$(\sin x)(\cos x)\left(\frac{\cos x}{\sin x}\right)$ | $= \cos^2 x$

$\cos^2 x$

2. $\cos x(\sec x - \cos x) = \sin^2 x$

$\cos x\left(\frac{1}{\cos x} - \cos x\right)$

Substitute

Expand

$\frac{\cos x}{\cos x} - \cos^2 x$

$1 - \cos^2 x$

Pythag. Ident

$\cos^2 x = 1 - \sin^2 x$

$1 - (1 - \sin^2 x)$

$1 - 1 + \sin^2 x$

$\sin^2 x$

$= \sin^2 x$

3. $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

common factor

$\cot^2 x [\csc^2 x - 1]$

$\cot^2 x [\cot^2 x]$

$\cot^4 x$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$(\cot^2 x)(\cot^2 x)$

$(\cot^2 x)[\csc^2 x - 1]$

$\cot^2 x \csc^2 x - \cot^2 x$

4. $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

fancy 1

common denominator

$$\frac{\sin x}{\cos x}\left(\frac{\sin x}{\sin x}\right) + \frac{\cos x}{\sin x}\left(\frac{\cos x}{\cos x}\right)$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$ ← ident.

$$\frac{1}{\cos x \sin x}$$

$$\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right) = \sec x \csc x$$ ≠

4. $\tan x + \cot x = \sec x \csc x$

$\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$

$\frac{1}{\cos x \sin x}$ ← Pythag Identity

$\frac{\cos^2 x + \sin^2 x}{\cos x \sin x}$

$\frac{\cos^2 x}{\cos x \sin x} + \frac{\sin^2 x}{\cos x \sin x}$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} =$

$\cot x + \tan x$

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#1-4