6.2

Sum, Difference, and Double-Angle Identities

Focus on...

- applying sum, difference, and double-angle identities to verify the equivalence of trigonometric expressions
- verifying a trigonometric identity numerically and graphically using technology

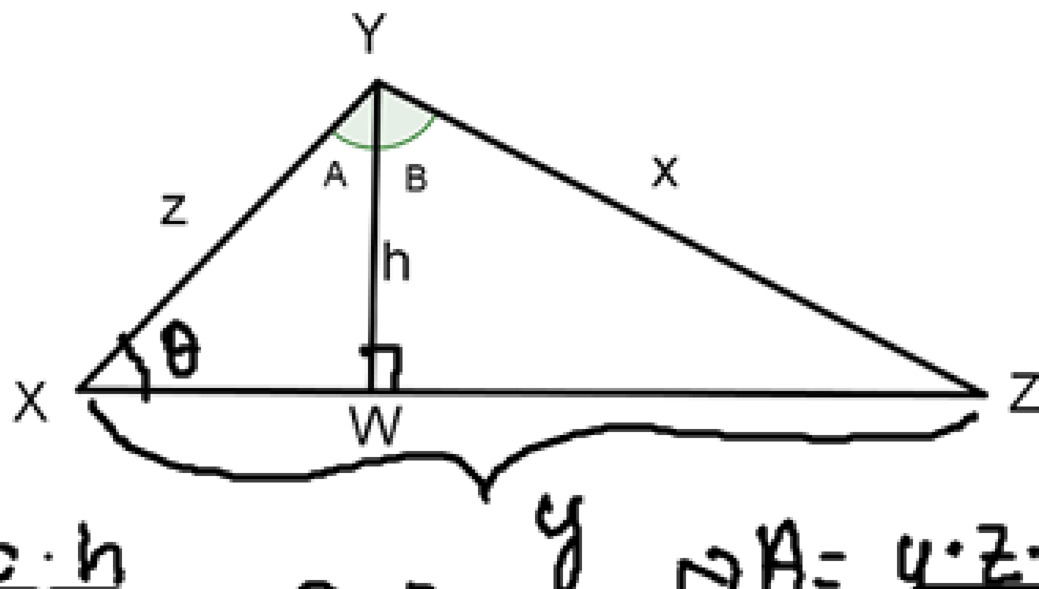
Sum & Difference Identities:

Sine:

Area of a triangle:

Per pendicular

$$A = \frac{b \times h}{2} \qquad \perp$$



$A = \frac{b \cdot h}{2}$

$A = \frac{y \cdot h}{2}$

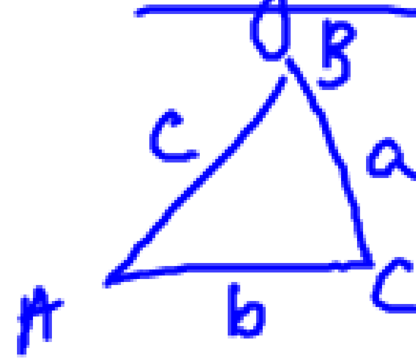
$\sin\theta = \frac{OPP}{HYP}$

$\sin X = \frac{h}{z}$

$h = z \cdot \sin X$

$\rightarrow A = \frac{y \cdot z \cdot \sin X}{2}$

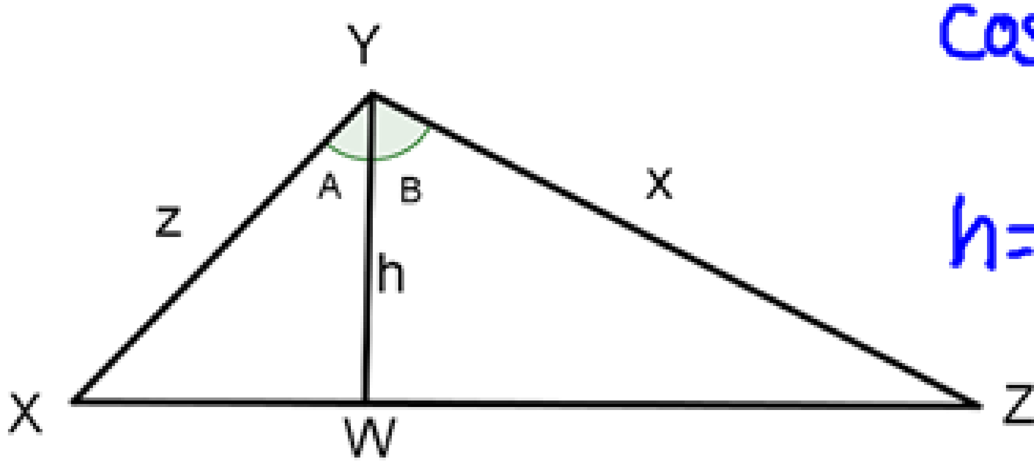
In general



$A = \frac{1}{2} ab \sin C$

Sum & Difference Identities:

Sine:



$$\cos\theta = \frac{ADJ}{HYP}$$

$$\cos A = \frac{h}{z} \qquad \cos B = \frac{h}{x}$$

$$h = z \cdot \cos A \qquad h = x \cdot \cos B$$

$$A_{\Delta XYZ} = A_{\Delta XYW} + A_{\Delta YWZ}$$

$$\frac{1}{2}xz\sin(A+B) = \frac{1}{2}zh\sin A + \frac{1}{2}xh\sin B$$

$$\frac{1}{2}xz\sin(A+B) = \frac{1}{2}z(x\cdot\cos B)\sin A + \frac{1}{2}x(z\cos A)\sin B$$

$$\frac{1}{2}xz\sin(A+B) = \frac{1}{2}xz\sin A\cos B + \frac{1}{2}xz\sin B\cos A$$

$$\frac{1}{\cancel{2}}\cancel{\times 2}\sin(A+B) = \frac{1}{\cancel{2}}\cancel{\times 2}\sin A\cos B + \frac{1}{\cancel{2}}\cancel{\times 2}\sin B\cos A$$

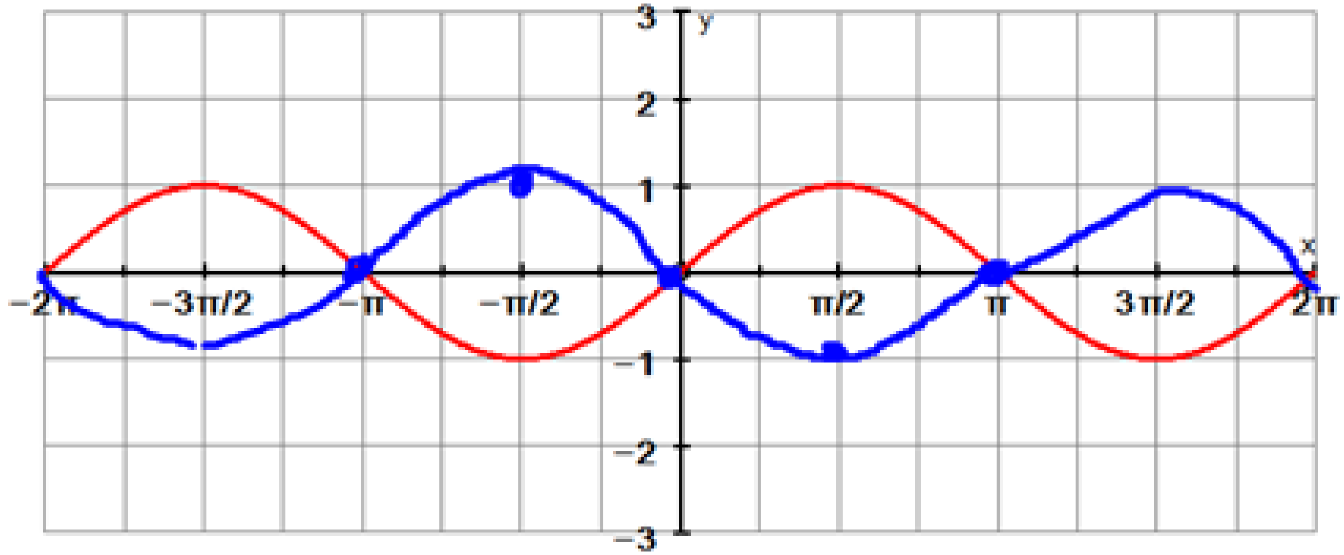
$$\boxed{\sin(A+B) = \sin A\cos B + \sin B\cos A}$$

$y = \sin(-x)$

y=sinx

R_y looks like a R_x

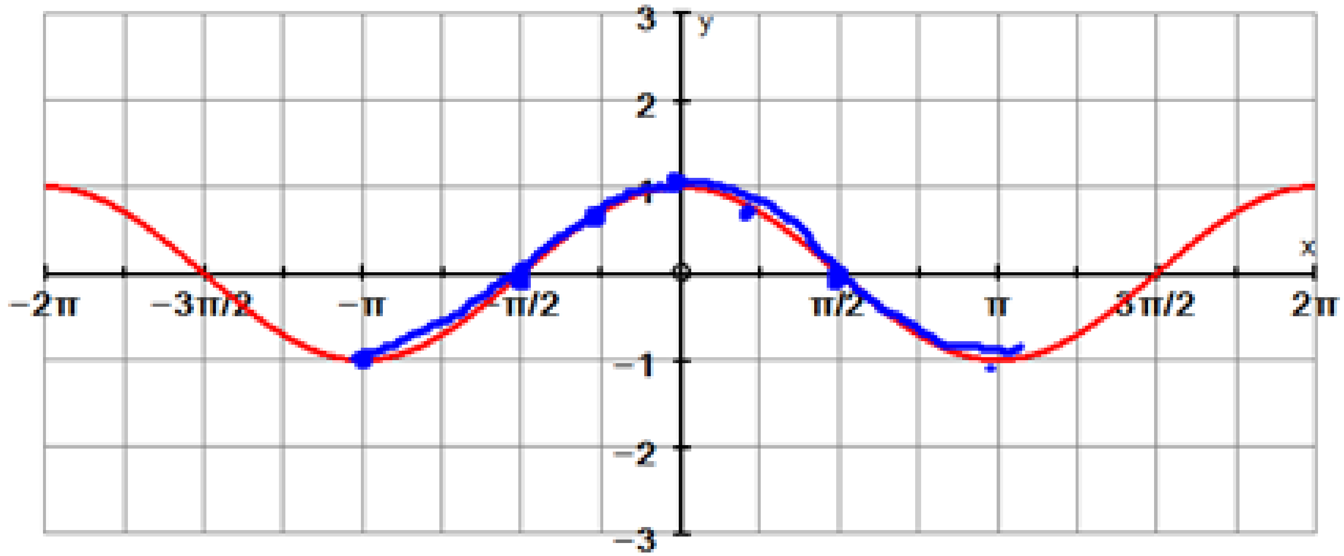
$\sin(-x) = -\sin x$



y=cosx

R_y is the same as a regular cosine curve

$\cos(-x) = \cos x$



To get $\sin(A-B)$ replace B with $-B$

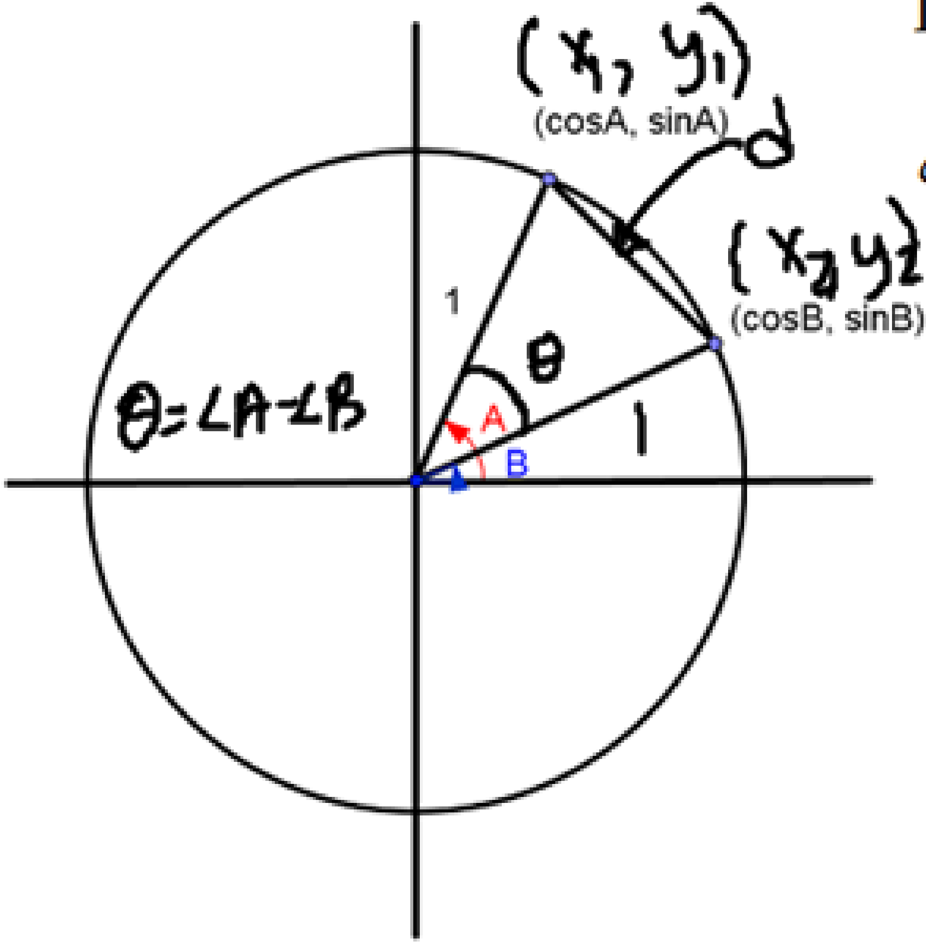
$$\sin(A-B) = \sin A\cos(-B) + \sin(-B)\cos A$$

$$\cos(-B) = \cos B$$

$$\sin(-B) = -\sin B$$

$$\boxed{\sin(A-B) = \sin A\cos B - \sin B\cos A}$$

Cosine:



Distance formula between two points:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d^2 = (\cos B - \cos A)^2 + (\sin B - \sin A)^2$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$d^2 = (1)^2 + (1)^2 - 2(1)(1)\cos(A-B)$$

$$d^2 = d^2$$

$$(\cos B - \cos A)^2 + (\sin B - \sin A)^2 = 2 - 2\cos(A - B)$$

$(a-b)^2$

$(a-b)(a-b)$

$a^2 - 2ab + b^2$

$$[\cos^2 B - 2\cos B\cos A + \cos^2 A] + [\sin^2 B - 2\sin B\sin A + \sin^2 A] = 2 - 2\cos(A-B)$$

☆ recall $\sin^2\theta + \cos^2\theta = 1$

$$2 - 2\cos B\cos A - 2\sin B\sin A = 2 - 2\cos(A-B)$$

☆ subtract 2 from both sides

$$-2\cos B\cos A - 2\sin B\sin A = -2\cos(A-B)$$

☆ Divide each term by -2

$$\boxed{\cos(A-B) = \cos A\cos B + \sin A\sin B}$$

replace B with $-B$

$$\cos(A-(-B)) = \cos A\cos(-B) + \sin A\sin(-B)$$

$$\boxed{\cos(A+B) = \cos A\cos B - \sin A\sin B}$$

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

Sum and Difference Identities:

$$\sin(A+B)=\sin A\cos B+\sin B\cos A$$

$$\sin(A-B)=\sin A\cos B-\sin B\cos A$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

Examples:

1. Simplify. Write the following expression as a single trigonometric function.

$$\sin 32° \cos 19° - \cos 32° \sin 19°$$

let $A = 32°$

$B = 19°$

$= \sin A \cos B - \cos A \sin B$

$= \sin(A - B)$

$= \sin(32° - 19°)$

$= \sin(13°)$

2. Calculate the exact value for:

(a) $\cos 15° = \cos(45° - 30°)$

$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} + \sqrt{2}}{4}$$

Calculate the exact value for:

(b) $\csc\left(\frac{5\pi}{12}\right)$

$$= \csc\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$$

$$= \csc\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \frac{1}{\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

$$\sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \sin\frac{\pi}{6}\cos\frac{\pi}{4} + \cos\frac{\pi}{6}\sin\frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$= \frac{4}{\sqrt{2}+\sqrt{6}}\left(\frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}\right)$$

$$= \frac{4\sqrt{2}-4\sqrt{6}}{(\sqrt{2})^2-(\sqrt{6})^2} = \frac{4\sqrt{2}-4\sqrt{6}}{2-6}$$

$$= \frac{4\sqrt{2}-4\sqrt{6}}{-4}$$

$$= -\sqrt{2}+\sqrt{6}$$

HW: pg 306 #1abd,2abd,8