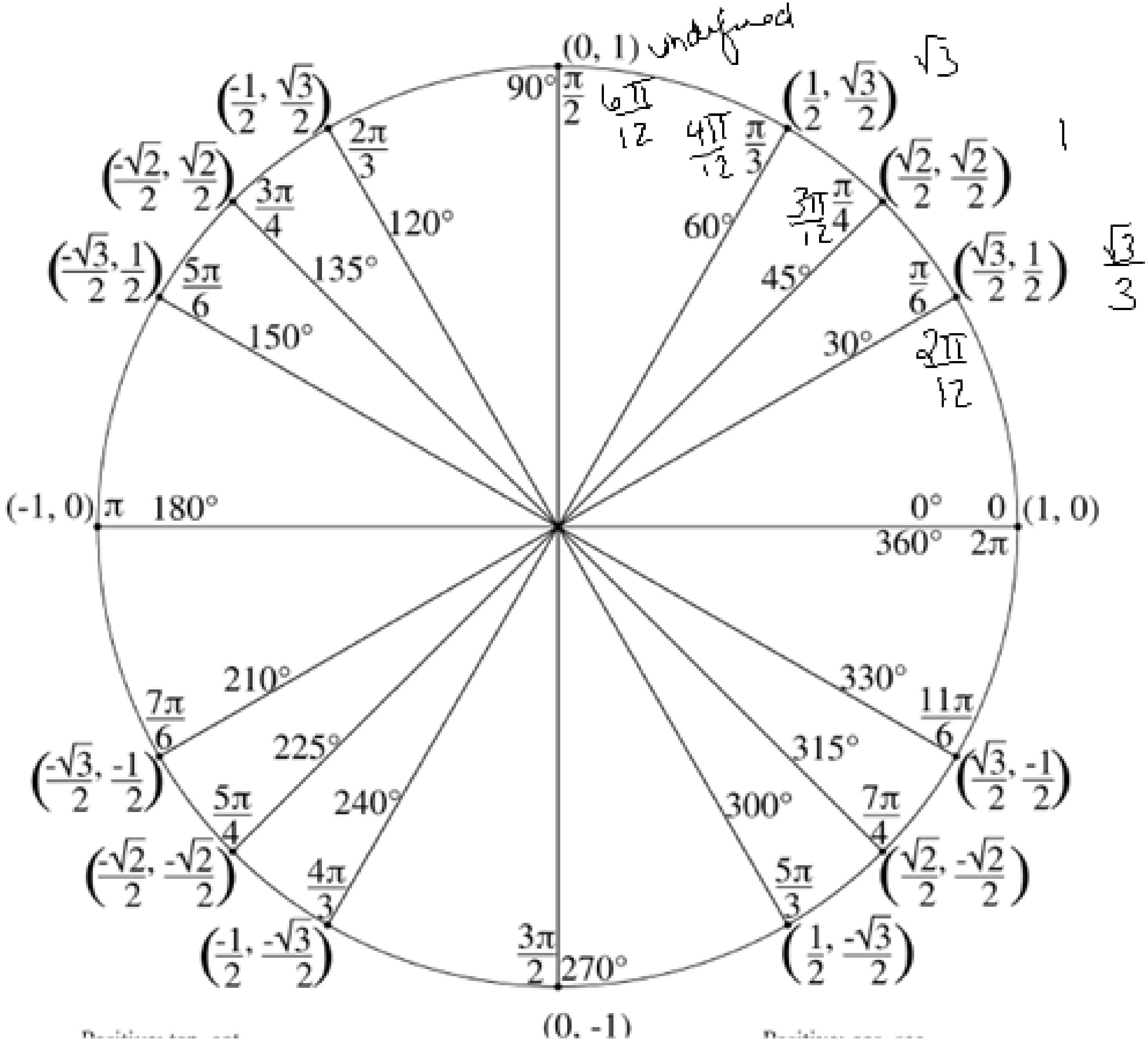

(0, 1) undefined
90° $\frac{\pi}{2}$
$\frac{6\pi}{12}$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\sqrt{3}$
$\frac{4\pi}{12}$ $\frac{\pi}{3}$
60°
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
1
$\frac{3\pi}{12}$ $\frac{\pi}{4}$
45°
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\sqrt{3}}{3}$
$\frac{\pi}{6}$
30°
$\frac{2\pi}{12}$
$\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ $\frac{2\pi}{3}$ 120°
$\left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$ 135°
$\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{5\pi}{6}$ 150°
(-1, 0) π 180°
0° 0 (1, 0)
360° 2π
210° $\frac{7\pi}{6}$ $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$
225° $\frac{5\pi}{4}$ $\left(\frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
240° $\frac{4\pi}{3}$ $\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$
$\frac{3\pi}{2}$ 270°
(0, -1)
300° $\frac{5\pi}{3}$ $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$
315° $\frac{7\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$
330° $\frac{11\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

Example:

Find the exact value of $\tan\left(\frac{5\pi}{12}\right)$

Double-Angle Identities

1. Use the identity $\sin(A+B)=\sin A\cos B+\cos A\sin B$ to create an identity for $\sin(2A)$.

$$\sin(2A)=\sin(A+A)$$
$$=\sin A\cos A+\cos A\sin A$$

$$\boxed{\sin 2A=2\sin A\cos A}$$

2. Use the identity $\cos(A+B)=\cos A\cos B-\sin A\sin B$ to create an identity for $\cos(2A)$.

$\cos 2A = \cos(A+A)$

$= \cos A\cos A - \sin A\sin A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

3. Use the Pythagorean identity $\sin^2 A+\cos^2 A=1$ to write an identity for $\cos(2A)$ that contains only the cosine ratio.

$\sin^2 A = 1-\cos^2 A$

$\cos 2A = \cos^2 A - [1-\cos^2 A]$

$\boxed{\cos 2A = 2\cos^2 A - 1}$

4. Write an identity for $\cos(2A)$ that contains only the sine ratio.

$\cos^2 A = 1 - \sin^2 A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = [1 - \sin^2 A] - \sin^2 A$

$$\boxed{\cos 2A = 1 - 2\sin^2 A}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\boxed{\tan(2A) = \frac{2\tan A}{1-\tan^2 A}}$$

Examples:

Given that $\sin\theta = -\frac{5}{13}$ and $\cos\theta = \frac{12}{13}$, find $\tan 2\theta$.

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\frac{-5}{13}}{\frac{12}{13}}$$

$$\tan\theta = \frac{-5}{12}$$

$$\tan 2\theta = \frac{\cancel{2}\left(\frac{-5}{\cancel{12}\,6}\right)}{1-\left(\frac{-5}{12}\right)^2}$$

$$= \frac{\frac{-5}{6}}{\frac{144}{144}-\frac{25}{144}}$$

$$= \frac{\frac{-5}{6}}{\frac{119}{144}}$$

$$\tan 2\theta = \frac{-5}{6} \div \frac{119}{144}$$

$$= \frac{-5}{6} \times \frac{144}{119}$$

$$= \frac{-5(\cancel{6}\cdot 2\cdot 12)}{\cancel{6}\cdot 119}$$

$$\tan 2\theta = \frac{-120}{119}$$

Your Turn

Consider the expression $\frac{\sin 2x}{\cos 2x + 1}$.

a) What are the permissible values for the expression?

b) Simplify the expression to one of the three primary trigonometric functions.

sinx, cosx, tanx

c) Verify your answer from part b), in the interval $[0, 2\pi)$, using technology.

A) everything But the non permissible values

$\cos 2x + 1 \neq 0$

$\cos 2x \neq -1$

$2x \neq \pi + 2\pi k, k \in I$

$x \neq \frac{\pi}{2} + \pi k, k \in I$

Super important

HW: pg 306 #1, 2, 4, 5, 7, 11, $\underbrace{14, 15, 16, 18, 20}$

$$\frac{\sin 2x}{\cos 2x + 1}$$

$$= \frac{2\sin x\cos x}{[2\cos^2 x - 1] + 1}$$

$$= \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \frac{\cancel{2}\sin x\cancel{\cos x}}{\cancel{2}\cos x\cancel{\cos x}}$$

$$= \frac{\sin x}{\cos x} = \tan x$$

$$\frac{\sin 2x}{\cos 2x + 1}$$

$$= \frac{2\sin x\cos x}{[2\cos^2 x - 1] + 1}$$

$$= \frac{2\sin x\cos x}{2\cos^2 x}$$

$$= \frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos x\,\cancel{\cos x}} = \frac{\sin x}{\cos x}$$

$$= \tan x$$