# 6.1

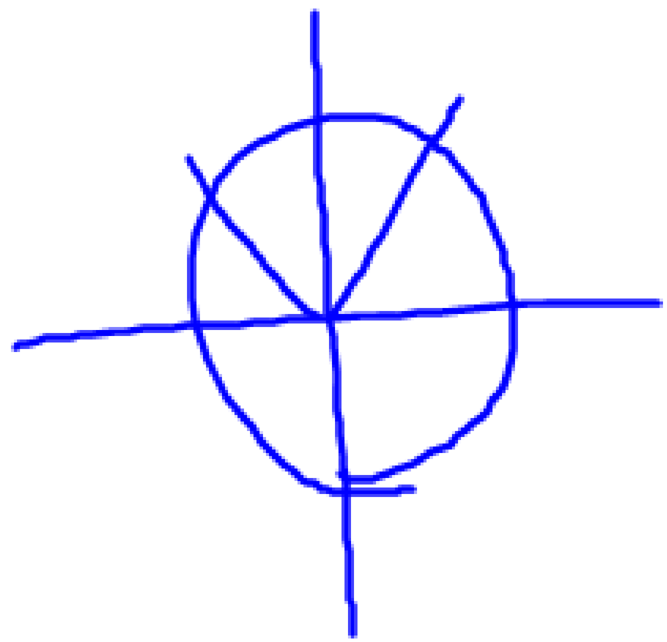
# Reciprocal, Quotient, and Pythagorean Identities

## Focus on...

- verifying a trigonometric identity numerically and graphically using technology
- exploring reciprocal, quotient, and Pythagorean identities
- determining non-permissible values of trigonometric identities
- explaining the difference between a trigonometric identity and a trigonometric equation

A trigonometric equation is an equation that is true only for certain values of $x$.

$$\sin x = \frac{1}{2}$$



Degrees

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \begin{cases} 30^\circ \\ 150^\circ \end{cases}$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2 \quad x = -3$

A trigonometric identity is an equation that is true for all permissible values of $x$ on both sides of the equation.

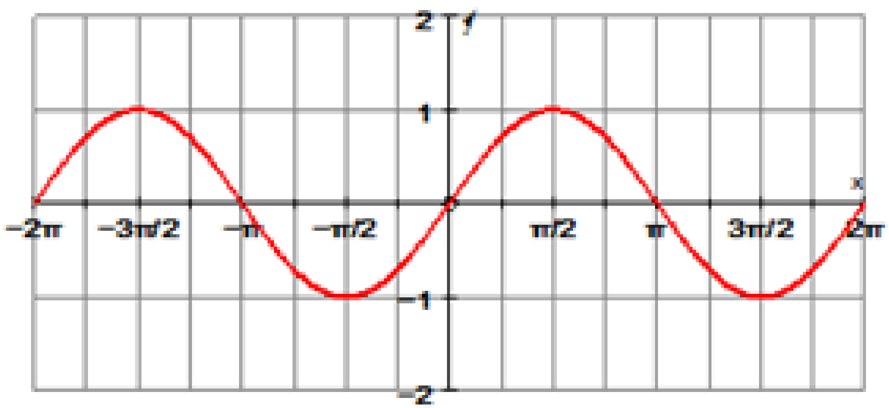
For example:

$$\sin x = \cos x \tan x$$

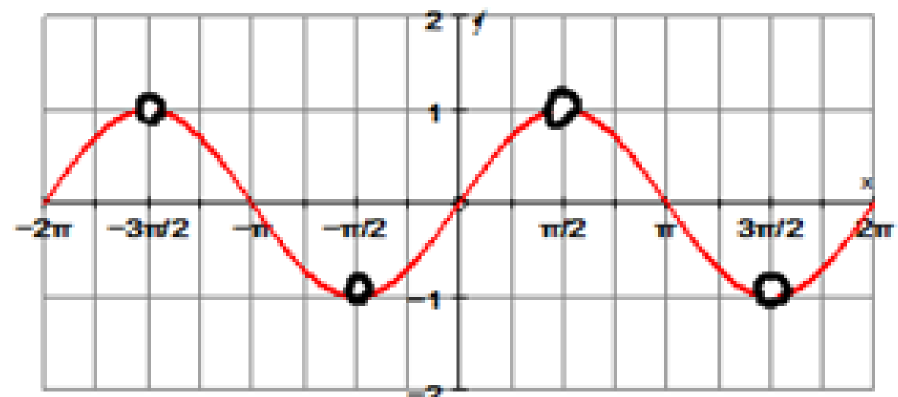
(a) Graph the curves $y = \sin x$ and $y = \cos x \tan x$. What do you notice?

Non permissible values

NPV




$y = \sin x$




$y = \cos x \tan x$

(b) What are the non-permissible values of $x$ in the equation $\sin x = \cos x \tan x$?

look for hidden denominators

$\sin x = \cos x \left(\frac{\sin x}{\cos x}\right)$

we don't want $\cos x = 0$

$x = \cos^{-1}(0)$

$x = \begin{cases} \frac{\pi}{2} \\ \frac{3\pi}{2} \end{cases} + 2\pi k, k \in I$

(c) Verify that 60° and $x = \frac{\pi}{4}$ are solutions of the equation.

$\sin 60° \stackrel{?}{=} \cos 60° (\tan 60°)$

$\frac{\sqrt{3}}{2} \stackrel{?}{=} \left(\frac{1}{2}\right)\left(\sqrt{3}\right)$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ✓

$\sin \frac{\pi}{4} \stackrel{?}{=} \cos \frac{\pi}{4} \left(\tan \frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2} \stackrel{?}{=} \frac{\sqrt{2}}{2}(1)$

$\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$ ✓

## Reciprocal Identities:

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x} \qquad \tan x = \frac{1}{\cot x}$$

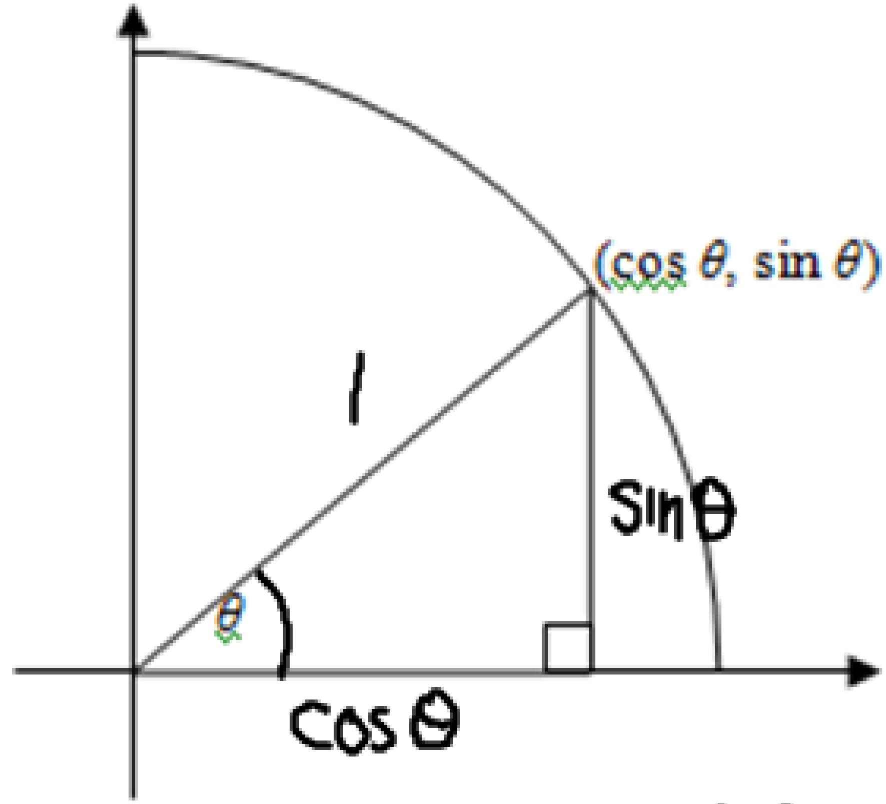
## Quotient Identities:

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

# Pythagorean Identities:

$$a^2+b^2=c^2$$

$$[\cos\theta]^2+[\sin\theta]^2=[1]^2$$




$$\boxed{\cos^2\theta+\sin^2\theta=1}$$

$$[\cos 0]^2+[\sin 0]^2 \stackrel{?}{=} 1$$

$$1^2+0^2=1$$

✓

$$[\cos 787^\circ]^2+[\sin 787^\circ]^2 \stackrel{?}{=} 1$$

$$(0.15267)+(0.8473) \stackrel{?}{=} 1$$

$$1=1$$

Divide every term in the identity $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$. Simplify.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\left(\frac{\sin\theta}{\cos\theta}\right)^2 + \left(\frac{\cos\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Divide every term in the identity $\sin^2\theta + \cos^2\theta = 1$ by $\sin^2\theta$. Simplify.

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

The three forms of the Pythagorean identity are:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left.\begin{array}{l} \cot^2\theta + 1 = \csc^2\theta \\[2em] 1 + \tan^2\theta = \sec^2\theta \end{array}\right\} \text{have NPVs}$$

## Extend

**15.** Given $\csc^2 x + \sin^2 x = 7.89$, find the value of $\frac{1}{\csc^2 x} + \frac{1}{\sin^2 x}$.

$$= \frac{1}{\csc^2 x}\left(\frac{\sin^2 x}{\sin^2 x}\right) + \frac{1}{\sin^2 x}\left(\frac{\csc^2 x}{\csc^2 x}\right)$$

$$= \frac{\sin^2 x + \csc^2 x}{\csc^2 x \sin^2 x}$$

$$= \frac{7.89}{\csc^2 x \sin^2 x}$$

$$\frac{1}{2} + \frac{1}{3}$$

fancy one

$$\frac{1}{2}\left(\frac{3}{3}\right) + \frac{1}{3}\left(\frac{2}{2}\right)$$

$$\frac{3}{6} + \frac{2}{6}$$

$$\frac{3+2}{6}$$

$$= \frac{7.89}{\csc^2 x \sin^2 x}$$

$$= \frac{7.89}{\left(\frac{1}{\cancel{\sin^2 x}}\right) \cdot \cancel{\sin^2 x}}$$

$$= 7.89$$

$$= \frac{1}{\csc^2 x} + \frac{1}{\sin^2 x}$$

$$= \sin^2 x + \csc^2 x$$

$$= 7.89$$

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