Which one doesn't Belong?

2	$\frac{19\pi}{6}$ $\frac{7\pi}{6}$ 210°	1110° Q1 30°	1 $\begin{array}{r} 1110° \\ -360° \\ \hline \end{array}$
3	30° $-\frac{11\pi}{6}$	$\frac{7\pi}{3}$ $\begin{array}{r} 420° \\ -360 \\ \hline 60° \end{array}$	4

4.2

The Unit Circle

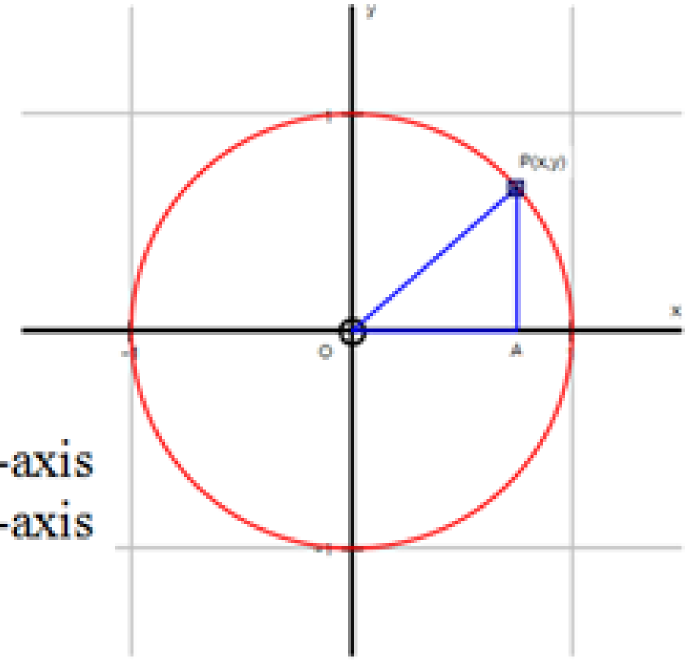
Focus on...

- developing and applying the equation of the unit circle
- generalizing the equation of a circle with centre (0, 0) and radius r
- using symmetry and patterns to locate the coordinates of points on the unit circle

The Equation of the Unit Circle:

The unit circle is a circle with radius 1 and centre at the origin.

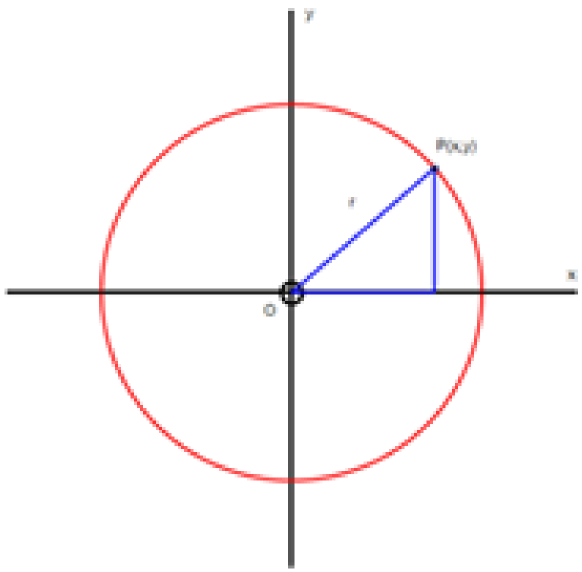
Point P is a point, (x, y), on the unit circle. A right angle triangle, OPA, can be constructed.



$OP = 1$	definition of unit circle
$PA = \lvert y \rvert$	distance from the point to the x-axis
$OA = \lvert x \rvert$	distance from the point to the y-axis
$(OP)^2 = (OA)^2 + (PA)^2$	Pythagorean Theorem

The equation of any circle, radius r, with centre at the origin is:

$x^2 + y^2 = r^2$

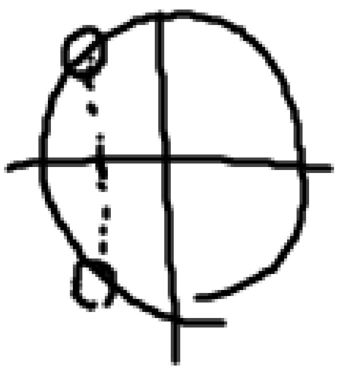


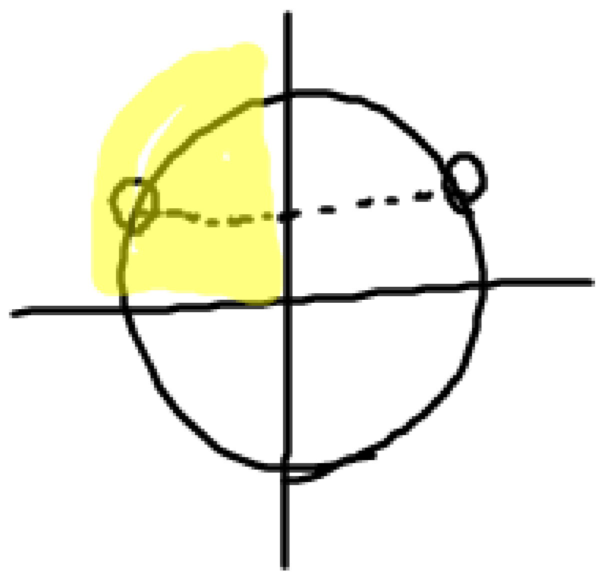
Example: Determine the equation of a circle with centre at the origin and radius 5.

Your Turn

Determine the missing coordinate(s) for all points on the unit circle satisfying the given conditions. Draw a diagram and tell which quadrant(s) the points lie in.

a) $\left(-\frac{5}{8}, y\right)$ **b)** $\left(x, \frac{5}{13}\right)$, where the point is in quadrant II

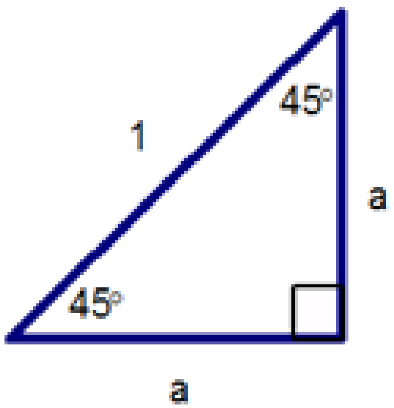




Special Angles:

θ = 45°

For a right angled triangle, if θ = 45°, then the triangle will be isosceles with two equal sides.



Use the Pythagorean Theorem to find the length of side a.

$$a^2 + b^2 = c^2$$

$$a^2 + a^2 = 1^2$$

$$2a^2 = 1$$

$$a^2 = \frac{1}{2}$$

$$\sqrt{a^2} = \sqrt{\frac{1}{2}}$$

$$a = \pm\frac{1}{\sqrt{2}}$$ ← rationalize fancy "1"

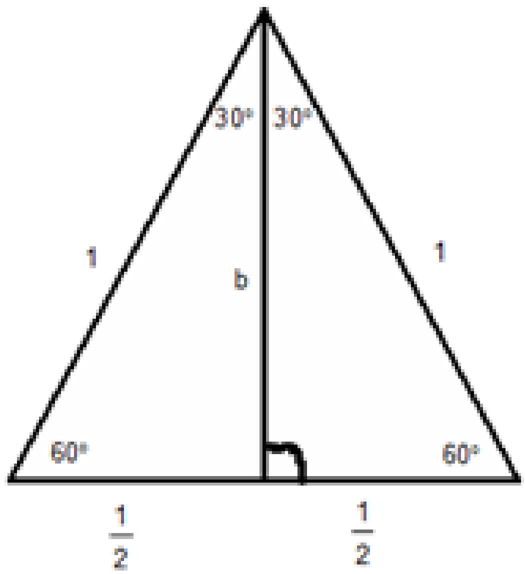
$$a = \pm\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$a = \pm\frac{\sqrt{2}}{2}$$ ← can't have a negative side length

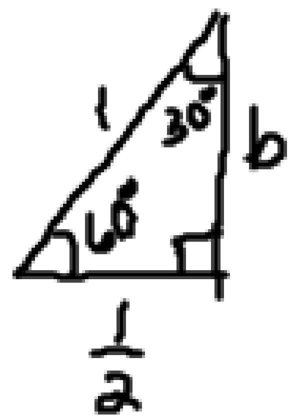
$$a = \frac{\sqrt{2}}{2}$$

θ = 60°

The triangle is half of an equilateral triangle:



Use the Pythagorean Theorem to find the length of side b:



$$a^2 + b^2 = 1^2$$

$$\left(\frac{1}{2}\right)^2 + b^2 = 1$$

$$b^2 = \frac{4}{4} - \frac{1}{4}$$

$$b^2 = \frac{3}{4}$$

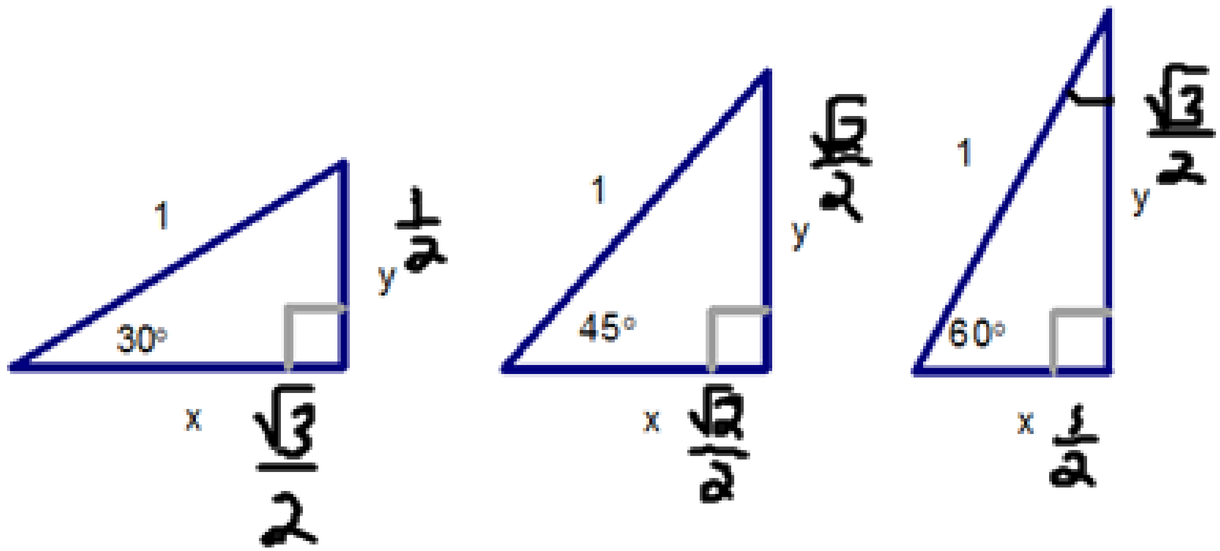
$$\sqrt{b^2} = \sqrt{\frac{3}{4}}$$

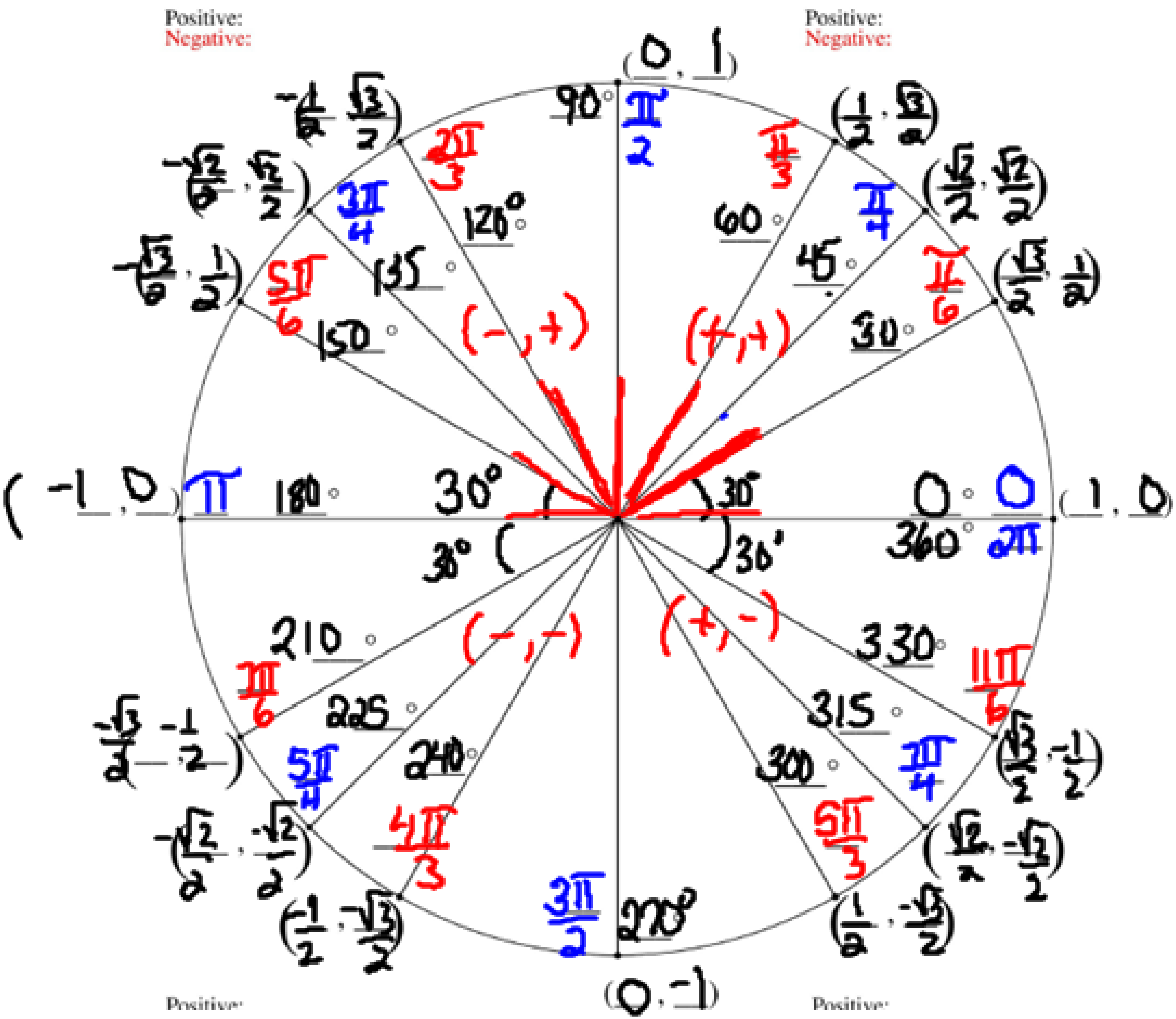
$$= \pm\frac{\sqrt{3}}{2}$$

reject negative

$$b = \frac{\sqrt{3}}{2}$$

Find the value for x and y for each of the following triangles:




Positive:
Negative:
Positive:
Negative:
(0, 1)
90°
π/2
(1/2, √3/2)
π/3
60°
(√2/2, √2/2)
π/4
45°
(√3/2, 1/2)
π/6
30°
(+,+)
(1, 0)
0°
0
360°
2π
30°
30°
(−1/2, √3/2)
2π/3
120°
(−√2/2, √2/2)
3π/4
135°
(−√3/2, 1/2)
5π/6
150°
(−,+)
(−1, 0)
π
180°
30°
30°
210°
7π/6
(−√3/2, −1/2)
225°
5π/4
(−√2/2, −√2/2)
240°
4π/3
(−1/2, −√3/2)
(−,−)
3π/2
270°
(0, −1)
330°
11π/6
(√3/2, −1/2)
315°
7π/4
(√2/2, −√2/2)
300°
5π/3
(1/2, −√3/2)
(+,−)
Positive:
Positive:

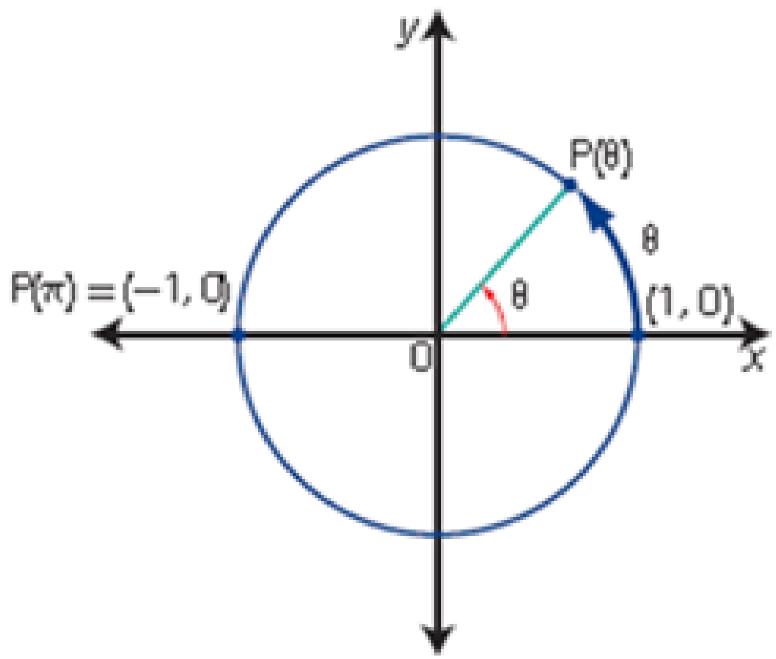
DON'T WRITE THIS ....IT'S IN THE BOOK

Radians and the Unit Circle:

arc length: $a = (\theta)(r)$

Since the unit circle has a radius of 1, $a = \theta$.

Use the function $P(\theta) = (x, y)$ to link arc length, θ, of a central angle to the coordinates (x, y) of the point of intersection of the terminal arm and the unit circle.

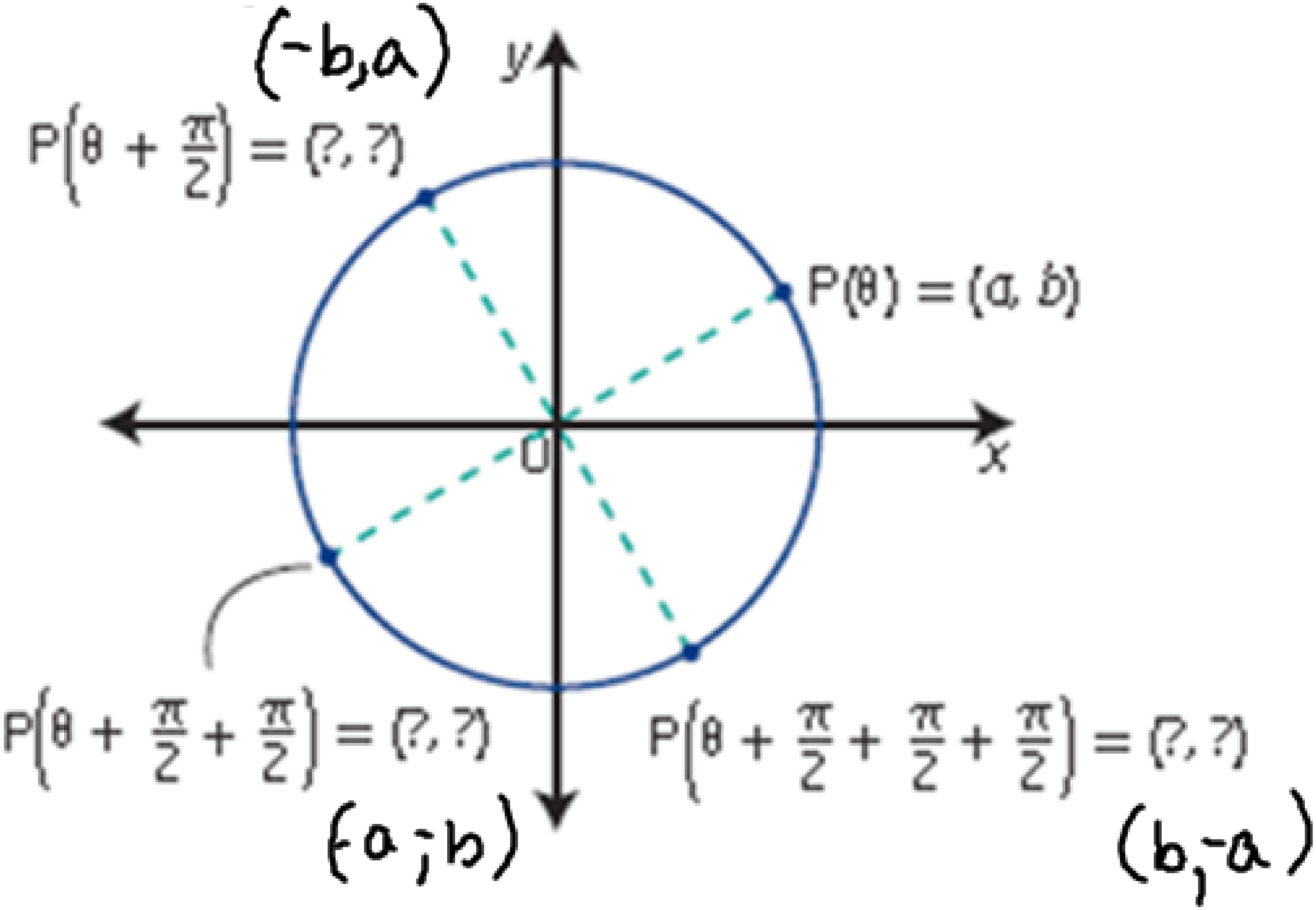


$P\left(\frac{\pi}{2}\right) = (0,1)$ $P\left(\frac{\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$P\left(-\frac{5\pi}{6}\right) = P\left(\frac{7\pi}{6}\right) = \left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$$

find coterminal angle

$$-\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{7\pi}{6}$$



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Point	Step 2: $+\frac{1}{4}$ turn	Step 3: $-\frac{1}{4}$ turn	Step 4: Description	Diagram
$P(0)=(1,0)$	$P\left(\frac{\pi}{2}\right)=$ $(0,1)$	$P\left(-\frac{\pi}{2}\right)=$ $P\left(\frac{3\pi}{2}\right)=$ $(0,-1)$		
$P\left(\frac{\pi}{3}\right)=$ $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ $(+,+)$	$P\left(\frac{\pi}{3}+\frac{\pi}{2}\right)=$ $\frac{2\pi}{6}+\frac{3\pi}{6}$ $P\left(\frac{5\pi}{6}\right)=\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$	$P\left(\frac{\pi}{3}-\frac{\pi}{2}\right)=$ $\frac{2\pi}{6}-\frac{3\pi}{6}=\frac{-\pi}{6}$ $P\left(-\frac{\pi}{6}\right)=P\left(\frac{11\pi}{6}\right)=\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	Switch x+y spots	

$P\left(\frac{5\pi}{3}\right)=$	$P\left(\frac{5\pi}{3}+\frac{\pi}{2}\right)=$	$P\left(\frac{5\pi}{3}-\frac{\pi}{2}\right)=$		

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13. If $P(\theta) = \left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$, determine the following.

a) What does $P(\theta)$ represent? Explain using a diagram.

b) In which quadrant does θ terminate?

c) Determine the coordinates of $P\left(\theta + \frac{\pi}{2}\right)$.

d) Determine the coordinates of $P\left(\theta - \frac{\pi}{2}\right)$.