4.1

Angles and Angle Measure

Focus on…

- sketching angles in standard position measured in degrees and radians
- converting angles in degree measure to radian measure and vice versa

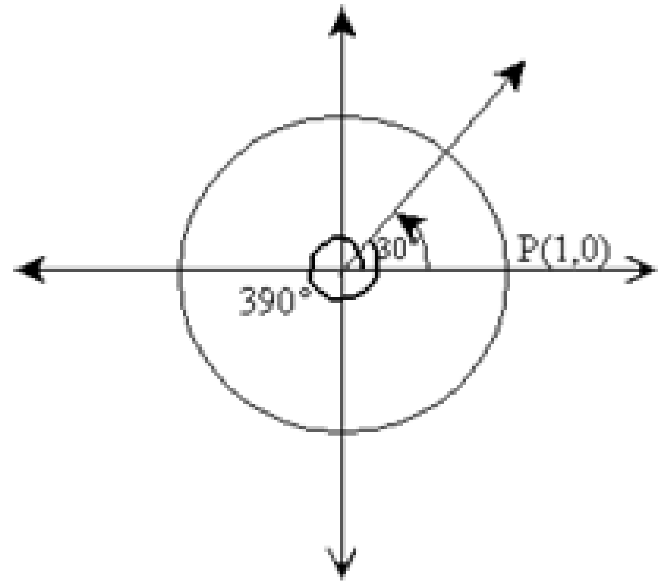
Co terminal Angles

Because of the periodic nature of circles, after one rotation through the circle, the values begin to repeat themselves.

Find the following values on the calculator:

$\sin 30° = 0.5$ $\cos 30° = 0.866$ $\frac{\sqrt{3}}{2}$

$\sin 390° = 0.5$ $\cos 390° = 0.866$



These two angles are said to be co terminal because they both terminate at the same terminal arm for the given angles.

- Every angle drawn will have 2 measures depending on the direction of measurement.
- If measured counterclockwise the angle is positive, if measured clockwise, the angle is negative:

Example: Determine one positive and one negative angle measure that is co terminal with each angle. In which quadrant does the terminal arm belong?

A) 256°

256° QIII

$256^\circ + 360^\circ = 616^\circ$

$256^\circ - 360^\circ = -104^\circ$

B) $\frac{3\pi}{4}$

$\frac{3\pi}{4} + 2\pi = \frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$

$\frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{5\pi}{4}$

In general, any angle θ could have an infinite number of co terminal angles with continued rotations. If we are using radians, we can still have co terminal angles: These are symbolized as:

In degrees: $\theta \pm 360°n,\ n \in N$ In radians: $\theta \pm 2\pi n,\ n \in N$

1 full rotation

Natural #s

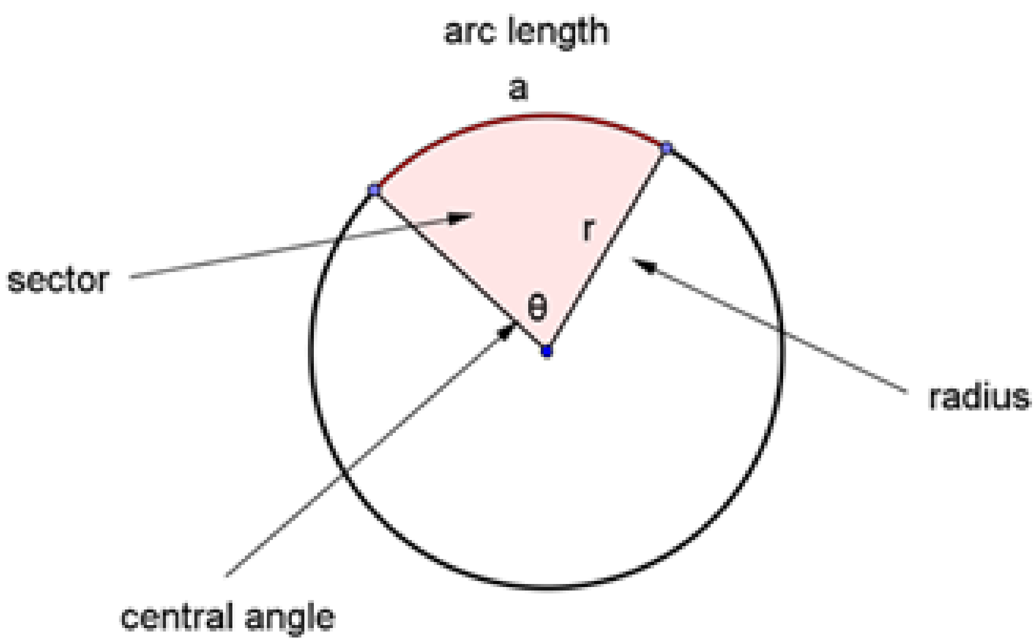
0, 1, 2, 3, 4, . . .

$\theta + 360° k,\ k \in I$

integers

. . . , -1, 0, 1 . . .

Arc Length of a Circle

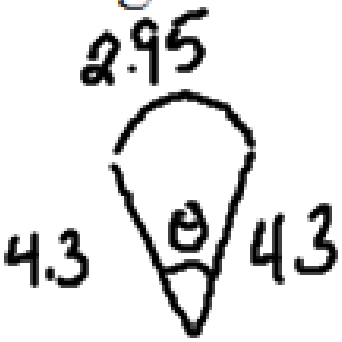


The **arc length**, a, of a circle with radius r, subtended by a central angle of θ, is given by:

$$a = (\theta)(r)$$

The angle θ is measured in radians, and a and r must be measured in the same units.

Example: Find the angle of a sector with a radius of 4.3 m and an arc length of 2.95 m



$a = \theta r$

$\theta = \dfrac{a}{r} = \dfrac{2.95\,m}{4.3\,m} = 0.6860465116$

$= 0.69$

Example: A sector has radius 5 cm and angle 55°. Find its arc length

Convert 55° to radian

$55°\left(\dfrac{\pi}{180°}\right) = \dfrac{55\pi}{180}$

$= \dfrac{11\pi}{36}$

$a = \theta r$

$= \left(\dfrac{11\pi}{36}\right)(5)$

$= \dfrac{55\pi}{36}\,cm$ ← exact

or $= 4.799655443$

$\sim 4.8\,cm$

4.2

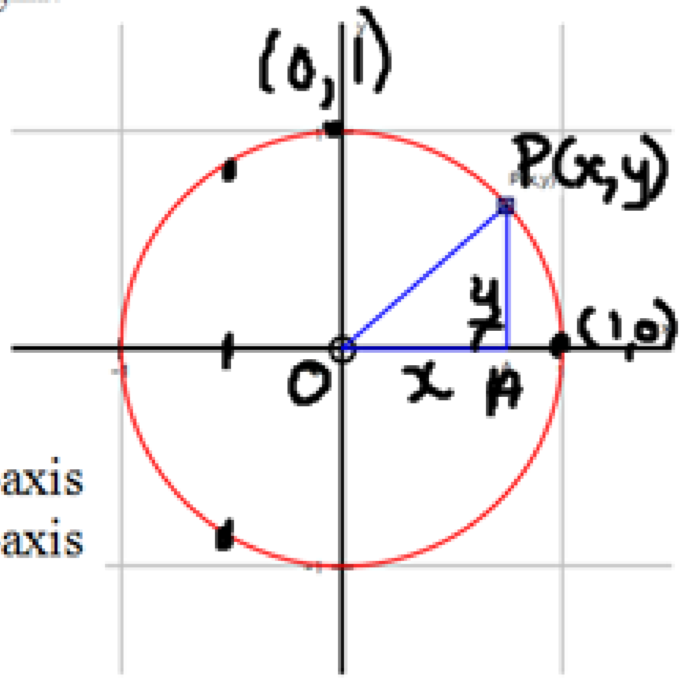
The Unit Circle

Focus on...

- developing and applying the equation of the unit circle
- generalizing the equation of a circle with centre (0, 0) and radius r
- using symmetry and patterns to locate the coordinates of points on the unit circle

The Equation of the Unit Circle:

The unit circle is a circle with radius 1 and centre at the origin.



Point P is a point, (x, y), on the unit circle. A right angle triangle, OPA, can be constructed.

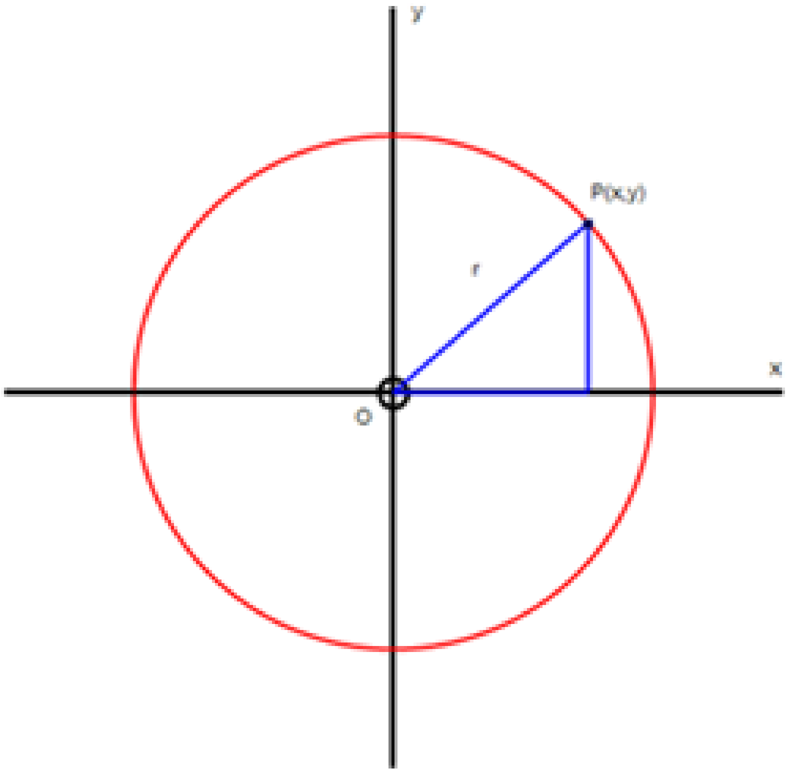
$OP = 1$	definition of unit circle
$PA = \|y\|$	distance from the point to the x-axis
$OA = \|x\|$	distance from the point to the y-axis
$(OP)^2 = (OA)^2 + (PA)^2$	Pythagorean Theorem

$1^2 = x^2 + y^2$ $\qquad$ $c^2 = a^2 + b^2$

$x^2 + y^2 = 1$

The equation of any circle, radius r, with centre at the origin is:

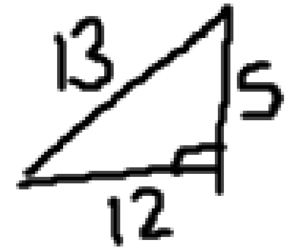
$x^2 + y^2 = r^2$



Example: Determine the equation of a circle with centre at the origin and radius 5.

$x^2 + y^2 = 5^2$

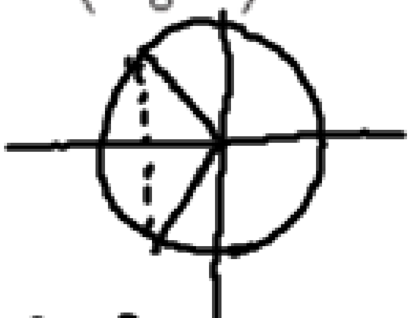
$x^2 + y^2 = 25$



Your Turn

Determine the missing coordinate(s) for all points on the unit circle satisfying the given conditions. Draw a diagram and tell which quadrant(s) the points lie in.

a) $\left(-\frac{5}{8}, y\right)$



$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y^2 = 1 - \left(-\frac{5}{8}\right)^2$

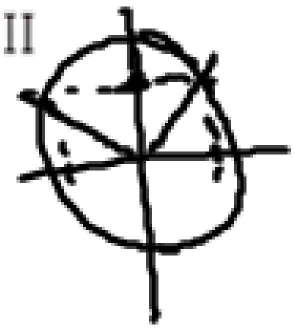
$y^2 = \frac{64}{64} - \frac{25}{64}$

$y = \pm\sqrt{\frac{39}{64}}$

$= \pm\frac{\sqrt{39}}{8}$

b) $\left(x, \frac{5}{13}\right)$, where the point is in quadrant II

$x \to$ negative QII



$x^2 = 1 - y^2$

$x^2 = 1 - \left(\frac{5}{13}\right)^2$

$= \frac{169}{169} - \frac{25}{169}$

$x^2 = \frac{144}{169}$

$x = \sqrt{\frac{144}{169}}$

$x = -\frac{12}{13}$

HW: pg 176 (4.1) #8-13, 16, 17 ← Big Must

everyother part

pg 186 #2 a, d, f