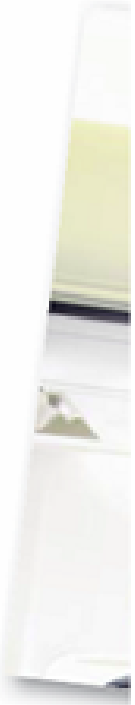
3.4

Equations and Graphs of Polynomial Functions



Focus on...

- describing the relationship between zeros, roots, and x-intercepts of polynomial functions and equations
- sketching the graph of a polynomial function without technology
- modelling and solving problems involving polynomial functions

Difference between **roots**, ***x*-intercepts**, and **zeros**:

Find the **roots** of the equation $3x^3 - 10x^2 - 23x - 10 = 0$

Find the **zeros** of the function $f(x) = 3x^3 - 10x^2 - 23x - 10$

Find ***x*-intercepts** of the graph of $f(x) = 3x^3 - 10x^2 - 23x - 10$

Repeated Roots:

$ax-b$

If a polynomial has a factor $x-a$ that is repeated n times, then $x=a$ is a zero of **multiplicity**, n.

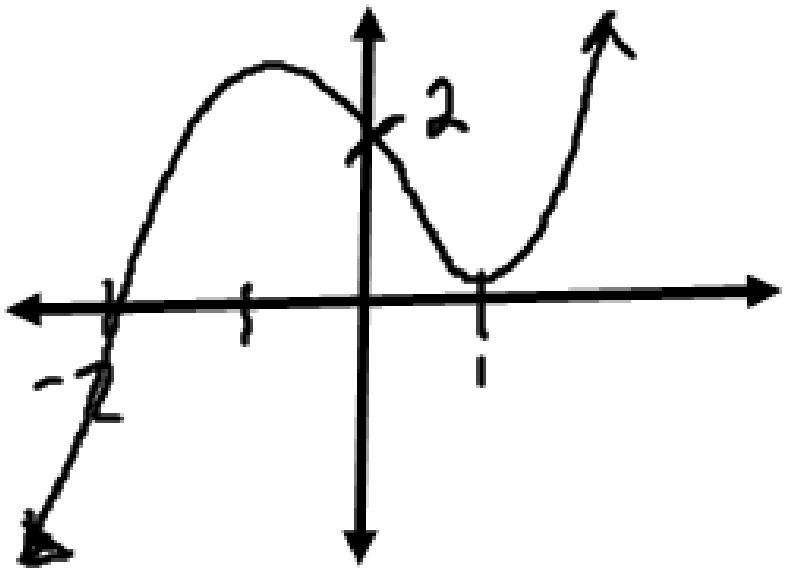
If the multiplicity of the zero is even, then the vertex touches the x-axis at $x=a$ (bounces)

$x=\frac{b}{a}$

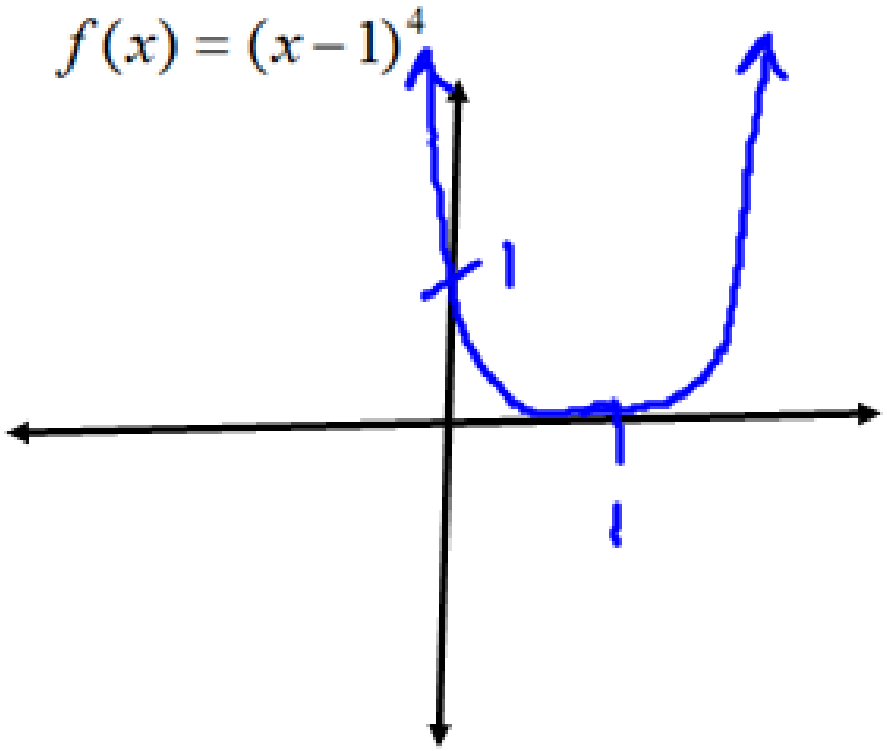
Example: $y=x^3$

$f(x)=(x-1)(x-1)(x+2)$

$=(x-1)^2(x+2)$



$f(x)=(x-1)^4$

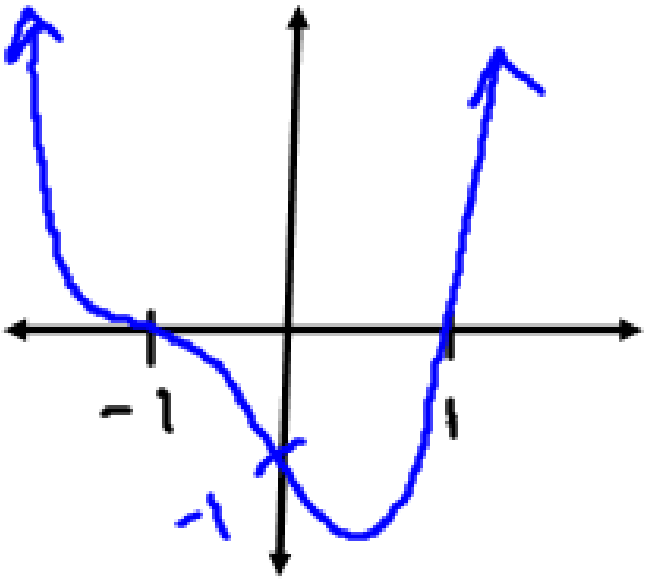


If the multiplicity of the zero is odd, then the function passes through the x-axis at $x = a$ (with a twist).

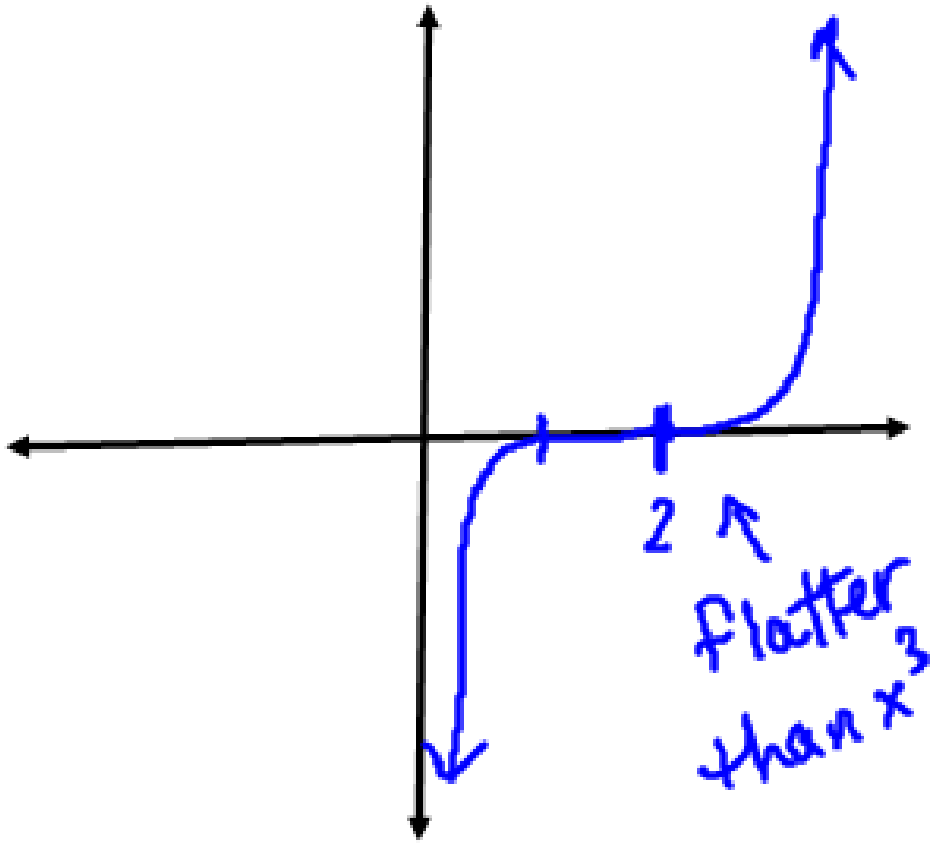
"attitude" inflection point

$f(x) = (x+1)(x+1)(x+1)(x-1)$

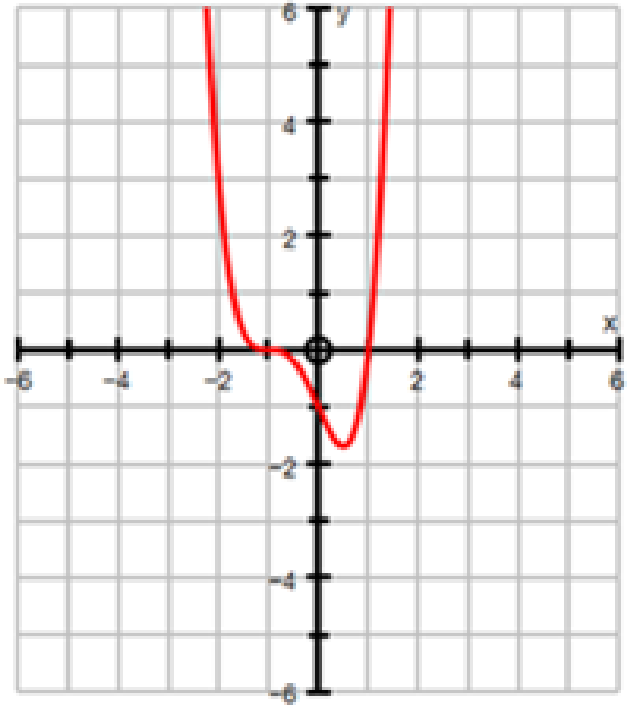
$f(x) = (x+1)^3(x-1)$



$f(x) = (x-2)^5$

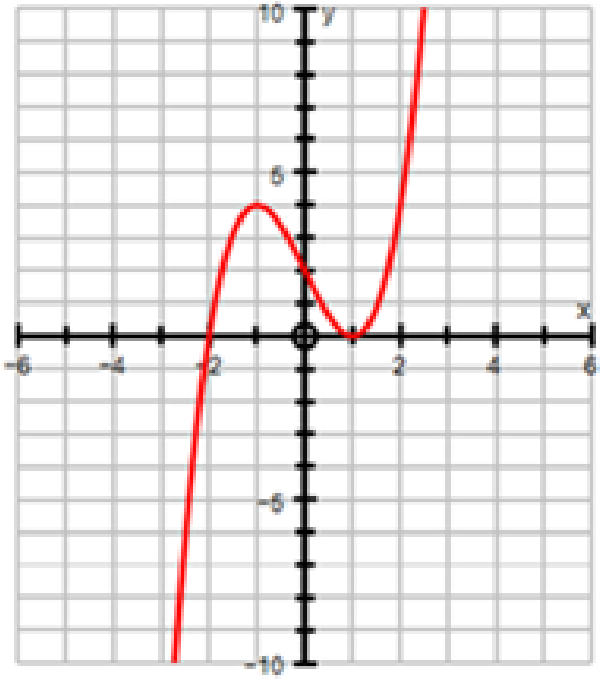


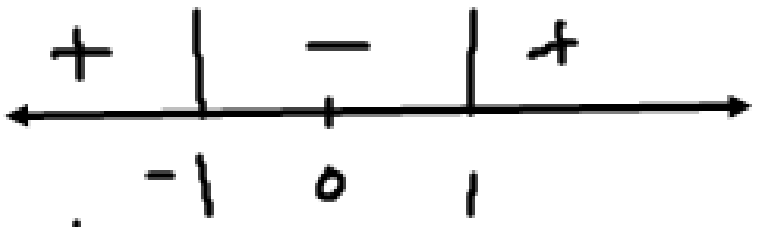
Sign diagrams and intervals (11PC)



+ if above x-axis

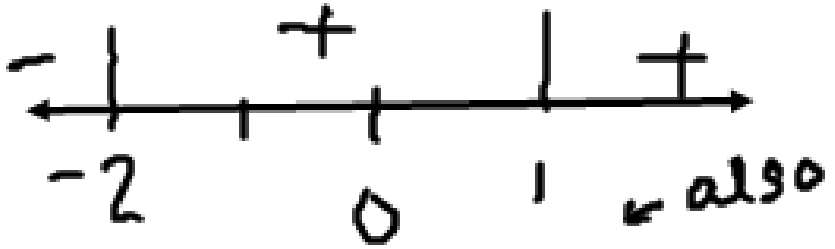
− if below x-axis





$f(x) > 0 : x \in (-\infty, -1) \cup (1, \infty)$

$f(x) < 0 : x \in (-1, 1)$

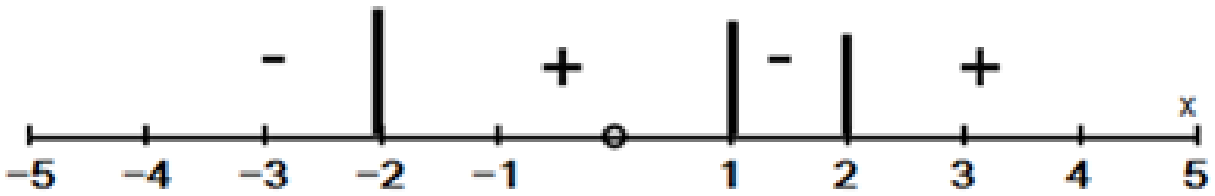


$f(x) > 0 : x \in (-2, 1) \cup (1, \infty)$

$f(x) < 0 : x \in (-\infty, -2)$

The following sign diagram was created from the polynomial $y = f(x)$

min. degree 3



Find the interval for which

A) $f(x) > 0$

$x \in (-2, 1) \cup (2, \infty)$

B) $f(x) \geq 0$

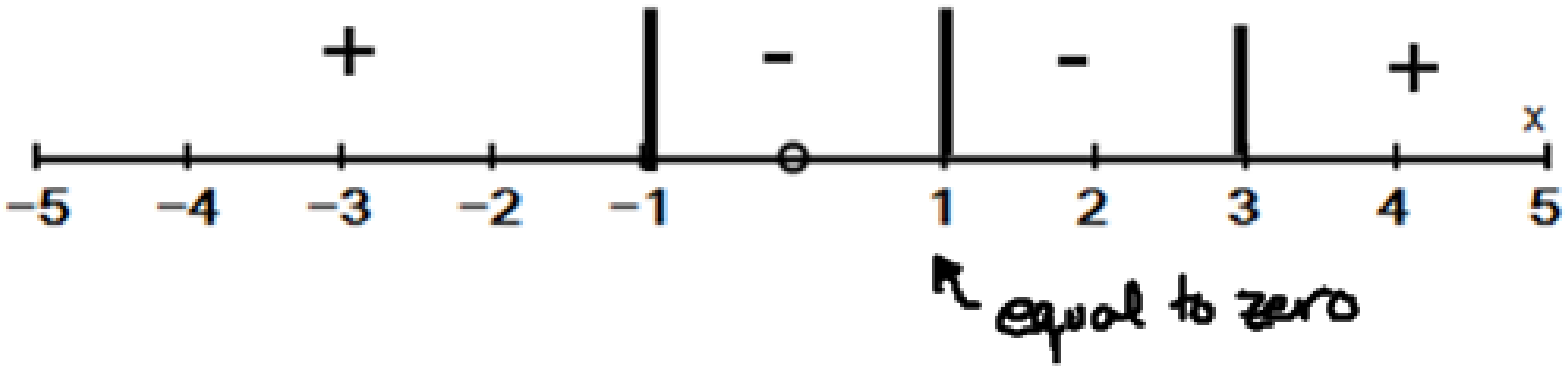
$x \in [-2, 1] \cup [2, \infty)$

C) $f(x) < 0$

$x \in (-\infty, -2) \cup (1, 2)$

D) $f(x) \leq 0$

$x \in (-\infty, -2] \cup [1, 2]$



Find the interval for which

A) $f(x) > 0$

$x \in (-\infty, -1) \cup (3, \infty)$

B) $f(x) \geq 0$

$x \in (-\infty, -1] \cup [1] \cup [3, \infty)$

C) $f(x) < 0$

$x \in (-1, 1) \cup (1, 3)$

D) $f(x) \leq 0$

$x \in [-1, 3]$

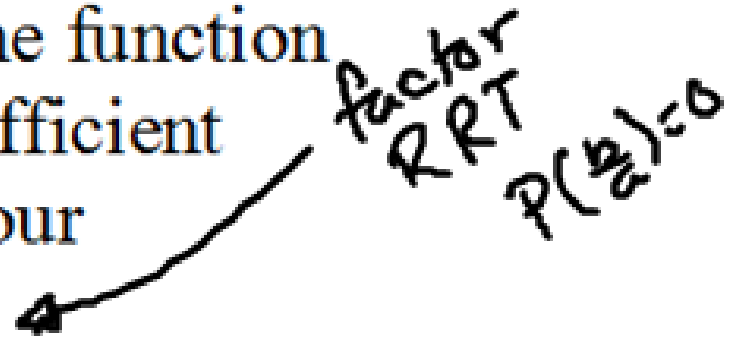
Analysing Graphs of Polynomial Functions:

- Identify the x and y-intercepts. Check for multiple roots.
- Determine the minimum degree of the polynomial.
- Use the end behaviour to determine the sign of the leading coefficient and to see if the polynomial is even or odd.
- Identify the intervals where the function is positive and where it is negative.

Sketching Graphs of Polynomial Functions
(without technology)

To sketch a graph you need to identify:

1. The degree of the function
2. The leading coefficient
3. The end behaviour
4. The zeros/x-int.
5. The y-int.
6. The intervals where the function is positive or negative.



$$f(x) = 4 - 3x^2 - x^3$$

$f(x) = -x^3 - 3x^2 + 4$

- degree 3
- end behaviour

start high
end low

- y-int (0, 4)
- x-int

$0 = -x^3 - 3x^2 + 4$

RRT

HW: pg 148
3-5, 9

3. $f(x) = -x^4 + 4x^3 - x^2 - 6x$

HW pg 148 # 3-5, 9