

Fantastically Fun Factoring Friday ☺

Simply Trinomial Factoring

$$\begin{aligned} & |x^2 - x - 72 \\ & (x - 9)(x + 8) \end{aligned} \quad \begin{aligned} & \underline{\quad} x \underline{\quad} = -72 \\ & \underline{\quad} + \underline{\quad} = -1 \end{aligned}$$

$$\begin{aligned} x^6 \cdot x^6 &= x^{6+6} \\ &= x^{12} \end{aligned}$$

Difference of squares

$$\begin{aligned} & x^2 - 121 \\ & (x + 11)(x - 11) \end{aligned}$$

$$\begin{aligned} & x^{12} - 16y^6 \\ & (x^6)^2 - (4y^3)^2 \\ & (x^6 - 4y^3)(x^6 + 4y^3) \end{aligned}$$

Decomposition

$6x^2 + 13x + 6$

1×6
 2×3

$6 \times 6 = 36$

$$\begin{aligned} -x - &= 36 \\ - + - &= 13 \end{aligned}$$

$$\underline{6x^2 + 9x} + \underline{4x + 6}$$

$$3x(2x + 3) + 2(2x + 3)$$

$$(2x + 3)(3x + 2)$$

2×1

$2x^2 - 13x - 7$

7×1

$$(2x + 1)(x - 7)$$

$$a = \pm 1, \pm 2$$

$$b = \pm 1, \pm 7$$

$$\frac{a}{b} = \pm 1, \pm 7, \pm \frac{1}{2}, \pm \frac{1}{7}$$

Ch 3.3 Day 2 - Specialized Factoring Techniques

Case 1 - Common Factors

A) $x^3 - x^2 - 12x$

$$x(x^2 - x - 12)$$

$$x(x - 4)(x + 3)$$

Discriminant

$$\Delta = b^2 - 4ac$$

if $\Delta < 0$ will not factor!

B) $2x^4 + 4x^3 + 4x^2 + 2x$

$$2x(x^3 + 2x^2 + 2x + 1)$$

Rational Root Theorem (RRT)

$$P\left(\frac{b}{a}\right) = 0$$

$$P(1) \neq 0$$

$$x = -1 \text{ is a root}$$

$$P(-1) = -1 + 2 - 2 + 1 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 2 & 1 \\ & \downarrow & -1 & -1 & -1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$$2x(x + 1)(x^2 + x + 1)$$

will not factor!

Case 2- The Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

A) $x^3 + 27$

$$(x)^3 + (3)^3$$

$$(x+3)((x)^2 - (3)(x) + (3)^2)$$

$$(x+3)(x^2 - 3x + 9)$$

B) $64x^3 + 125$

$$(4x)^3 + (5)^3$$

$$(4x+5)((4x)^2 - (4x)(5) + (5)^2)$$

$$(4x+5)(16x^2 - 20x + 25)$$

Case 2- The Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$c) 27x^3 - 1$$

$$(^{3x})^3 - (1)^3$$

$$(3x - 1)((3x)^2 + (3x)(1) + (1)^2)$$

$$(3x - 1)(9x^2 + 3x + 1)$$

Case 3 – Quartics factored as Quadratic trinomials

A) $x^4 - 5x^2 + 4$

let $m = x^2$

$$m^2 - 5m + 4$$

$$(m - 4)(m - 1)$$

$$(x^2 - 4)(x^2 - 1)$$

$$(x+2)(x-2)(x+1)(x-1)$$

B) $4x^4 - 37x^2 + 9$

let $m = x^2$

$$4m^2 - 37m + 9$$

$$4m^2 - 36m - m + 9$$

$$4m(m-9) - 1(m-9)$$

$$(m-9)(4m-1)$$

$$(x^2-9)(4x^2-1)$$

$$(x-3)(x+3)(2x-1)(2x+1)$$

$$\left(\begin{array}{l} -x \quad = 36 \\ + \quad = -37 \end{array} \right)$$

Case 4- Grouping to Find a Common Factor

$$\begin{aligned} \text{A) } & \underbrace{x^3 - 2x^2}_{x^2(x-2)} - \underbrace{16x + 32}_{16(x+2)} \\ & x^2(x-2) - 16(x+2) \\ & (x-2)(x^2 - 16) \\ & (x-2)(x-4)(x+4) \end{aligned}$$

$$\begin{aligned} \text{B) } & \overbrace{x^5 - 5x^4} - \overbrace{10x^3 + 50x^2} + \overbrace{9x - 45} \\ & x^4(x-5) - 10x^2(x-5) + 9(x-5) \\ & (x-5)(x^4 - 10x^2 + 9) \\ & \quad \text{let } x^2 = m \\ & (x-5)(m^2 - 10m + 9) \\ & (x-5)(m-9)(m-1) \\ & (x-5)(x^2-9)(x^2-1) \\ & (x-5)(x-3)(x+3)(x-1)(x+1) \end{aligned}$$

Factoring

1. Factor fully using the rational root theorem, or one of the special factoring techniques discussed in class.

a) $x^3 - 4x^2 + x + 6$

b) $2x^3 + x^2 - 13x + 6$

c) $x^3 + x^2 - 16x + 20$

d) $(x - 5)(3x^3 - x^2 - 20x - 12)$

e) $x^3 - 4x^2 - 4x + 16$

f) $x^3 - 64$

g) $5a^4 - 135a$

h) $x^3 - 5x^2 - 9x + 45$

i) $x^4 - 13x^2 + 36$

j) $x^4 + 8x^3 - 2x^2 - 16x$

k) $4x^3 - 8x^2 - 25x + 50$

l) $36x^2 - x^4 - 100$

m) $x^4 - x^3 - x + 1$

n) $x^5 - 2x^4 - x^3 + 2x^2 - 12x + 24$