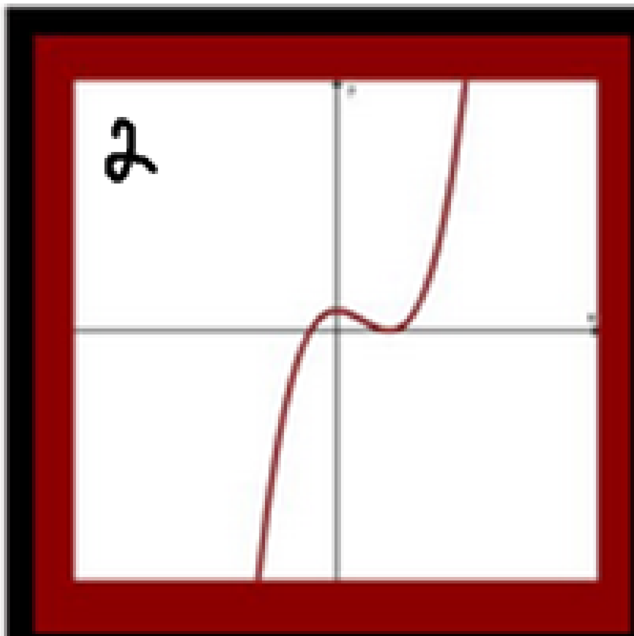
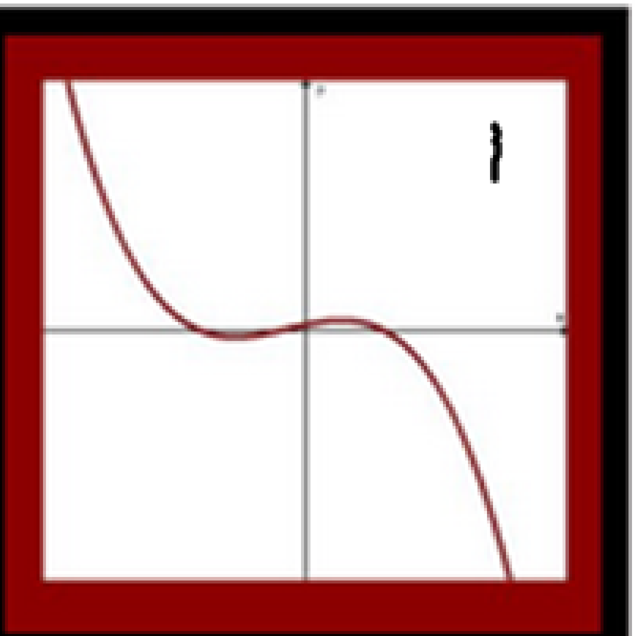
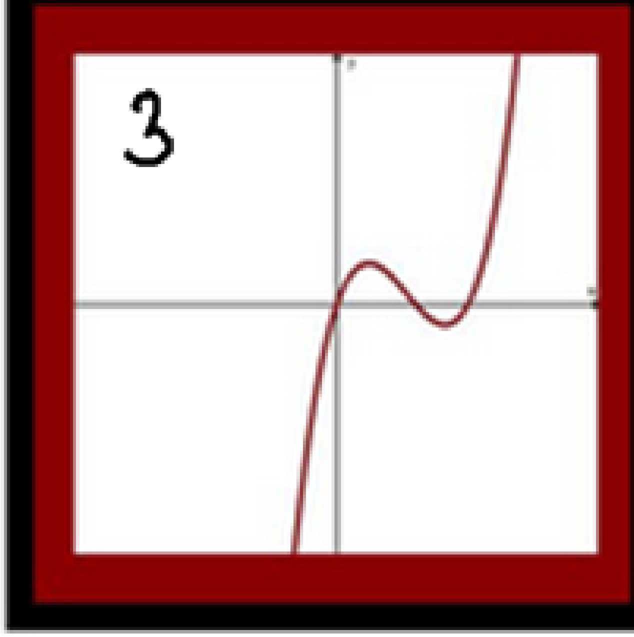
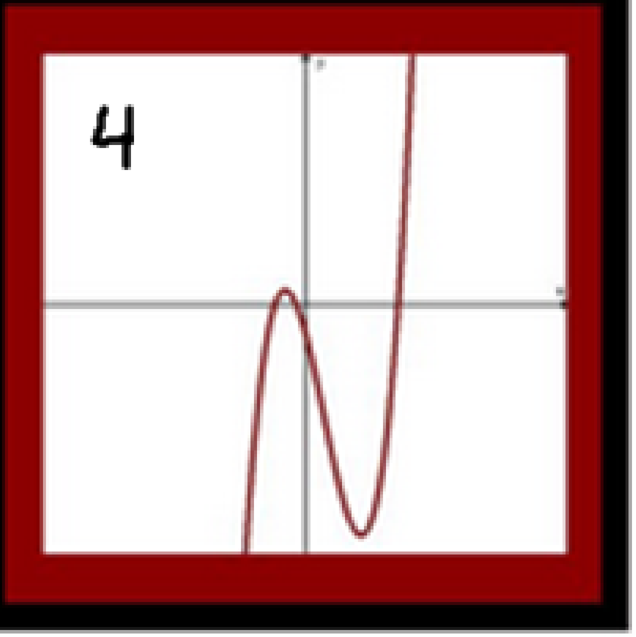
Which one doesn`t belong









Sketch the following;

$f(x) = -3(x-2)^2(x+1)^3(x-4)^1$

- degree : 6
- leading coeff: -3

  $a<0$ R_x
- End behaviour:

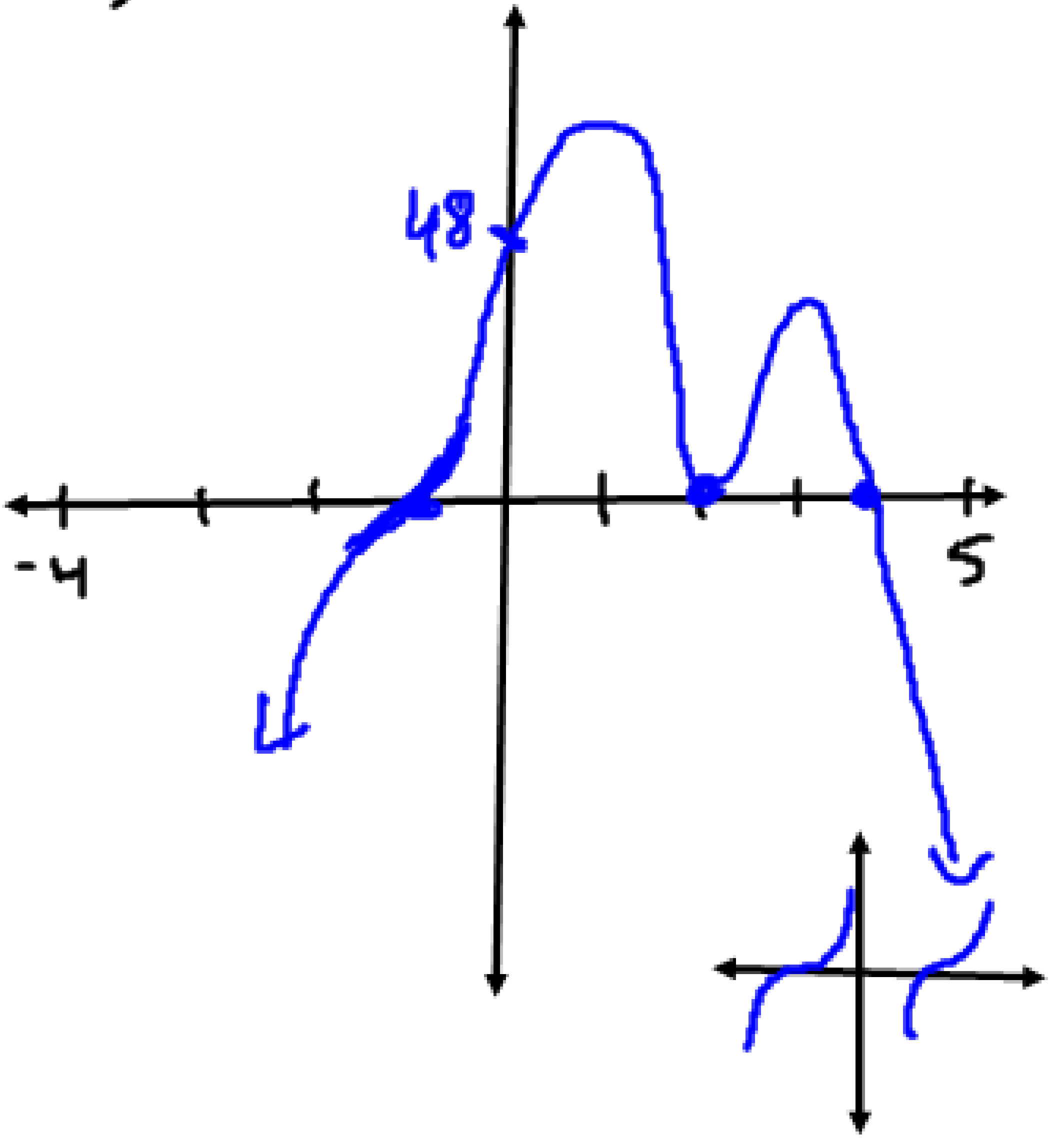
  start low

  end low
- x-int (y=0)

(2,0)	(-1,0)	(4,0)
↑	↑	↑
Bounce	attitude	cross

- y-int (x=0)

  $f(0) = -3(-2)^2(+1)^3(-4) = 48$ (0,48)



To try tonight (check with desmos)

$y = (x-5)^3(x+1)^2$

$y = -5(x-1)(x+1)(x-3)^2$

$y = -(x-4)(2-x)(x+3)^3$

$y = -(2x-3)(x-4)$

$y = -(x-5)(x+5)(x+1)$

3.3

The Factor Theorem

Focus on…

- factoring polynomials
- explaining the relationship between the linear factors of a polynomial expression and the zeros of the corresponding function
- modelling and solving problems involving polynomial functions

The Factor theorem states that a polynomial $P(x)$, has a factor $x-a$ $\underline{\text{if and only if}}$ (iff) $P(a)=0$

$\Leftrightarrow$

Example: $P(x)=x^5-x^4+x^3-2x+1$ has a factor of $x-1$

$$P(1) = (1)^5 - (1)^4 + (1)^3 - 2(1) + 1$$
$$= 1 - 1 + 1 - 2 + 1$$
$$= 0$$

$x-1$ is a factor

$x-1=0$

$x=1$ is a root

Integral Zero Theorem- if $x-a$ is a factor of a polynomial $P(x)$, then a is a factor of the constant term

$$P(x) = x^2 - 5x + 6$$
$$= (x-2)(x-3)$$

constant term is 6

1×6	-1×-6
2×3	-2×-3

↑

$$\_\times\_ = 6$$
$$\_ + \_ = -5$$

Example: What are the possible zeros of the following polynomial? $P(n) = n^3 - 3n^2 \underline{-10n +} 24$

$\underline{24}$

1×24	-1×-24
2×12	-2×-12
3×8	-3×-8
4×6	-4×-6

±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

The Integral Zero Theorem is a specialized case of the Rational Root Theorem

$(ax-b)$ is a factor of $P(x)$ if and only if (iff) $P\left(\frac{b}{a}\right)=0$.

$ax-b=0$

$ax=b$

$x=\frac{b}{a}$

Recall:

$$P(x)=\overset{a}{a_n}x^n+a_{n-1}x^{n-1}+\cdots+a_3x^3+a_{n-2}x^2+a_1x^1+\overset{b}{a_0}$$

constant term

leading coeffic.

So, b is the factors of a_0 and a is the factors of a_n

Example: write $P(x) = x^3 - 7x^2 - 4x + 28$ in factored form

Step 1- find the factors of the coefficient of the leading term and the constant term

$a = \pm 1$

$b = \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

$\frac{b}{a} = \pm 1, \pm 2, \pm 4, \pm 7, \pm 14, \pm 28$

↑ ↑

Step 2- calculate $P\left(\frac{b}{a}\right)$. If it equals zero, it is a rational root of $P(x)$.

$$P(2) = (2)^3 - 7(2)^2 - 4(2) + 28$$
$$= 8 - 28 - 8 + 28$$
$$= 0$$

Step 3- Using a root, create the factor $(ax-b)$ and factor polynomial.

x=2 is the root

x−2 is the factor

$$\begin{array}{c|cccc} 2 & 1 & -7 & -4 & 28 \\ & \downarrow & +2 & +-10 & +-28 \\ \hline x & 1 & -5 & -14 & 0 \end{array}$$

← remainder

Step 4- Repeat until fully factored ☺

$$\begin{aligned} P(x) &= x^3-7x^2-4x+28 \\ &= (x-2)(x^2-5x-14) \\ &= (x-2)(x-7)(x+2) \end{aligned}$$

Example: factor $g(x) = 4x^3 - 12x^2 + 5x + 6$

$\uparrow$ a $\uparrow$ b

$a = \pm 1, \pm 2, \pm 4$

$b = \pm 1, \pm 2, \pm 3, \pm 6$

$\frac{b}{a} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

$$P(-1) = 4(-1)^3 - 12(-1)^2 + 5(-1) + 6$$
$$= -4 - 12 - 5 + 6$$
$$\neq 0$$

$$P(2) = 4(2)^3 - 12(2)^2 + 5(2) + 6$$
$$= 32 - 48 + 10 + 6$$
$$= 0$$

$x = 2$ is a root

$x - 2$ is a factor

Example: factor $g(x) = 4x^3 - 12x^2 + 5x + 6$

$x = 2$

$$\begin{array}{c|cccc} 2 & 4 & -12 & 5 & 6 \\ & \downarrow & +8 & -8 & -6 \\ \hline x & 4 & -4 & -3 & 0 \end{array}$$

$g(x) = (x-2)(4x^2 - 4x - 3)$

$= (x-2)(\underbrace{4x^2 - 6x}_{} + \underbrace{2x - 3}_{})$

$= (x-2)(2x(2x-3) + 1(2x-3))$

$= (x-2)(2x-3)(2x+1)$

decomposition

$4x - 3 = -12$

$\_\_ \times \_\_ = -12$

$\_\_ + \_\_ = -4$

Example: find the degree 4 polynomial with x-int of (1,0), (-1,0), (-1/2,0), (-2, 0) and a y-int of (0, -6)

work backwards

$x=1$	$x=-1$	$x=-\frac{1}{2}$	$x=-2$
$x-1=0$	$x+1=0$	$x+\frac{1}{2}=0$	$x+2=0$
		$2x+1=0$	

$0=(x-1)(x+1)(2x+1)(x+2)$

$y=a(x-1)(x+1)(2x+1)(x+2)$

$-6=a(-1)(1)(1)(2)$

$-6=-2a$

$3=a$

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$$y = 3(x-1)(x+1)(2x+1)(x+2)$$

$$= 3(x^2-1)(2x^2+5x+2)$$

$$= 3(2x^4+5x^3+2x^2-2x^2-5x-2)$$

$$= 3(2x^4+5x^3-5x-2)$$

$$y = 6x^4+15x^3-15x+6$$

HW pg 133

#1-7