

3.2

The Remainder Theorem

Focus on...

- describing the relationship between polynomial long division and synthetic division
- dividing polynomials by binomials of the form $x - a$ using long division or synthetic division
- explaining the relationship between the remainder when a polynomial is divided by a binomial of the form $x - a$ and the value of the polynomial at $x = a$



Long division:

$$\begin{array}{r} 6215 \\ 9 \overline{) 55935} \\ \underline{-54} \downarrow \\ 19 \downarrow \\ \underline{-18} \downarrow \\ 13 \downarrow \\ \underline{-9} \downarrow \\ 45 \\ \underline{-45} \\ 0 \end{array}$$

$$\frac{55935}{9} = 6215$$

0 ← no remainder

$$\begin{array}{r} 635r3 \\ 12 \overline{) 7623} \end{array}$$

$$\begin{array}{r} \downarrow \\ \underline{-72} \downarrow \\ 42 \downarrow \\ \underline{-36} \downarrow \\ 63 \downarrow \\ \underline{-60} \\ 3 \end{array}$$

3 ← remainder

$$\begin{aligned} \frac{7623}{12} &= 635 + \frac{3}{12} \\ &= 635\frac{3}{12} \end{aligned}$$


$$(x-2)(?) = x^3 - 7x^2 - 4x + 28$$

$$(x-2)(x^2 - 5x - 14) = \overbrace{\hspace{1.5cm}}^{x^2 - 5x - 14}$$

Example 1: $x-2 \overline{) x^3 - 7x^2 - 4x + 28}$

$$\begin{array}{r} x^3 - 7x^2 - 4x + 28 \\ -(x^3 - 2x^2) \quad \downarrow \\ \hline 0 - 5x^2 - 4x \\ -(-5x^2 + 10x) \quad \checkmark \\ \hline 0 - 14x + 28 \\ -(-14x + 28) \\ \hline 0 \quad 0 \end{array}$$

$$\frac{x^3 - 7x^2 - 4x + 28}{x-2} = x^2 - 5x - 14$$

0 0 ← no remainder


$$3 \cdot ? = 36$$

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

Example 2: $x - 2 \overline{) x^3 - 2x^2 - 3x + 8}$

$x^2 + 0x - 3$ (Place holder)

$- (x^3 - 2x^2) \quad \downarrow \quad \downarrow$

$0 \quad 0 \quad -3x + 8$

$- (-3x + 6)$

$0 + 2 \leftarrow \text{remainder}$

$$\frac{x^3 - 2x^2 - 3x + 8}{x - 2} = x^2 - 3 + \frac{2}{x - 2}$$

→ The answer to a division Question.

3. Determine each quotient, Q , using long division.

e)
$$\frac{t^4 + 6t^3 - 3t^2 - t + 8}{t + 1}$$

$$\begin{array}{r} t^3 + 5t^2 - 8t + 7 \\ t+1 \overline{) t^4 + 6t^3 - 3t^2 - t + 8} \\ \underline{-(t^4 + t^3)} \downarrow \\ 0 + 5t^3 - 3t^2 \\ \underline{-(5t^3 + 5t^2)} \downarrow \\ 0 - 8t^2 - t \\ \underline{-(-8t^2 - 8t)} \downarrow \\ 0 + 7t + 8 \\ \underline{-(7t + 7)} \\ 0 + 1 \leftarrow \text{remainder} \end{array}$$

$$Q = t^3 + 5t^2 - 8t + 7$$

linear factor

two terms
↓

↓ When we divide a polynomial $P(x)$ by a binomial of the form $(x - a)$ $a \in I$, we write it as $\frac{P(x)}{x - a}$ and the result is

$\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$, where $Q(x)$ is the quotient and R is the remainder.

The Remainder Theorem:

When a polynomial $P(x)$, is divided by a binomial of the form $x - a$, $a \in I$, the remainder is $P(a)$

$$x - a = 0$$

$$x = a$$

Example 2 again...: $x - 2 \overline{) x^3 - 2x^2 - 3x + 8}$

$$P(x) = x^3 - 2x^2 - 3x + 8$$

$$x - 2 = 0$$

$$x = 2$$

$$P(2) = (2)^3 - 2(2)^2 - 3(2) + 8$$

$$= 8 - 2(4) - 6 + 8$$

$$= 8 - 8 - 6 + 8$$

$$= 2$$

Apply

8. For each dividend, determine the value of k if the remainder is 3.

a) $(x^3 + 4x^2 - x + k) \div (x - 1)$

$$\begin{aligned}x - 1 &= 0 \\x &= 1\end{aligned}$$

$$\begin{array}{r}x-1 \overline{) x^3 + 4x^2 - x + k} \\ \hline\end{array}$$

⋮

3 ← remainder

$$P(x) = x^3 + 4x^2 - x + k$$

$$P(1) = 3$$

$$3 = (1)^3 + 4(1)^2 - (1) + k$$

$$3 = 1 + 4 - 1 + k$$

$$3 = 4 + k$$

$$-1 = k$$

HW:
pg 124
1-3, 6, 8