3.1

Characteristics of Polynomial Functions

Focus on...

- identifying polynomial functions
- analysing polynomial functions

Equation	Name	Degree
$y = 2x^0$	constant	0
$y = 2x^1 + 1$	linear	1
$y = 3x^2 + 3x + 1$	Quadratic	2
$y = 4x^3 - 2x^2 + 3x + 1$	Cubic	3
$y = x^4 - x^3 - 4x^2 + 5x + 1$	Quartic	4
$y = x^3 - x^5$	Quintic	5

The degree of the polynomial comes from the highest exponent.

In general a polynomial function has the form:

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_{n-3} x^{n-3} + \ldots + a_0 x^0 \qquad , n \in N \quad a \in R$$

Natural

whole #s

include zero

0, 1, 2, 3, 4,

real

6

0.5

$-2, \frac{1}{4}$

$\sqrt{27}, \pi$

$\sqrt[3]{5}$ $\sqrt[3]{-7}$

$y = ax^2 + bx + c$

$y = a_2 x^2 + a_1 x + a_0$

$(-2)^3 = -8$

Examples: is it a polynomial?

A) $y = 3x^{-4}$ No

$y = \frac{3}{x^4}$

B) $y = 3\sqrt{x}$ $= 3x^{\frac{1}{2}}$ No

C) $y = 3^x$ -No

D) $y = (3x - 2)(x^2 - 1)$

Yes degree 3

E) $y = \frac{x^2 + 9}{2x - 7}$ No

Yes F) $y = \frac{x^2 + 9}{4} = \frac{1}{4}x^2 + \frac{9}{4}$

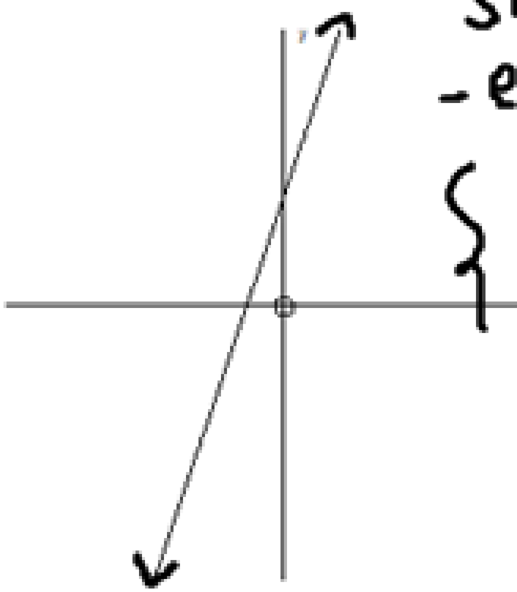
G) $y = \sqrt{3}x^7$ Yes

degree 7

What do these graphs look like??

<u>Polynomial of degree 1</u>: $y = ax + b$

- Straight line, one x-intercept
- If "a" is positive

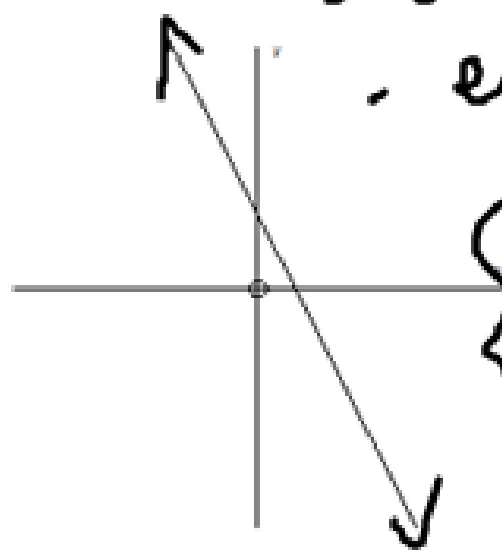


- start low
- end high

start QIII
end QI

Equation: y=3x+4

If "a" is negative



- start high
- end low

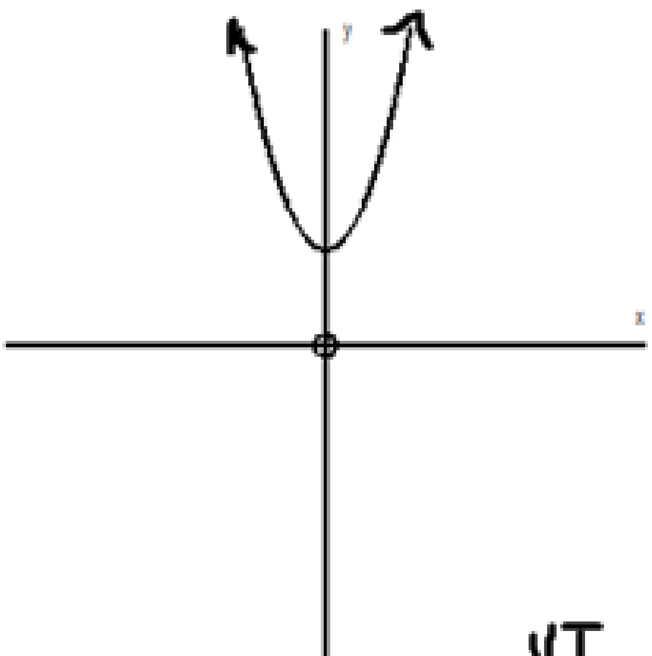
start QII
end QIV

Equation: y=–2x+3

Polynomial of degree 2: $y = ax^2 + bx + c$

- Quadratic
- If "a" is positive, graph opens up
- If "a" is negative, graph opens down

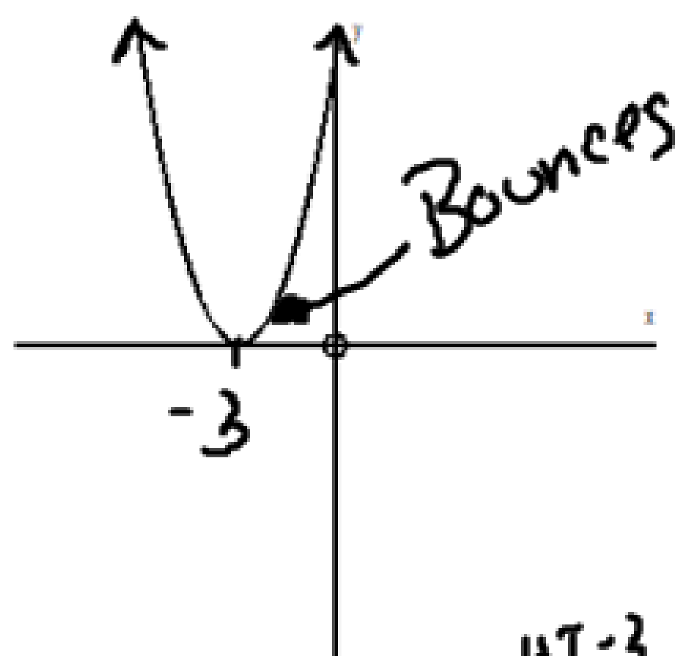
NO x-int (Complex roots)



VT

Equation: $y=x^2+3$

ONE x-int (double root)

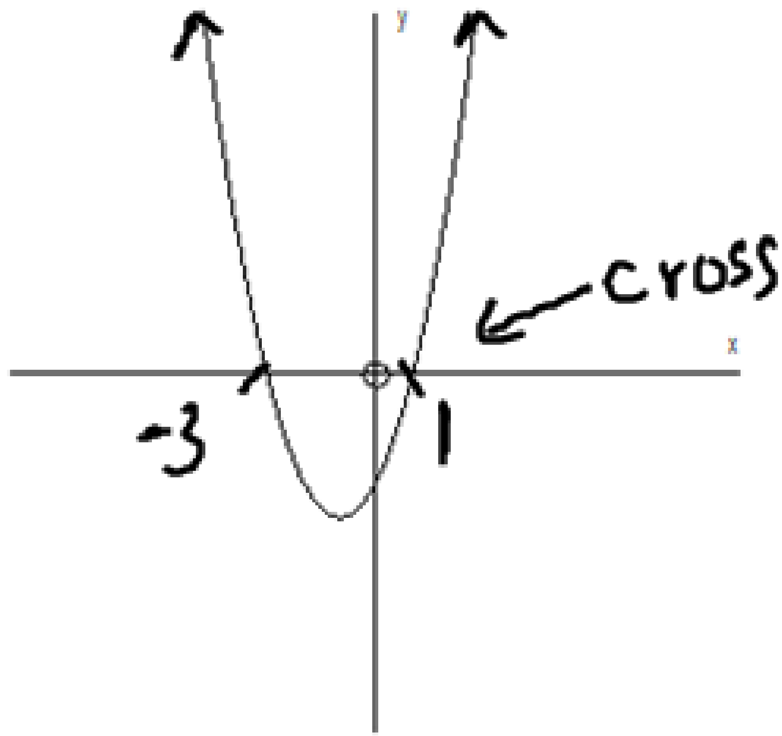


HT -3

Equation: $y=(x+3)^2$

$(x+3)(x+3)$

TWO x-int (real and distinct roots)



Equation: $y=(x+3)(x-1)$

- Start high
- end high

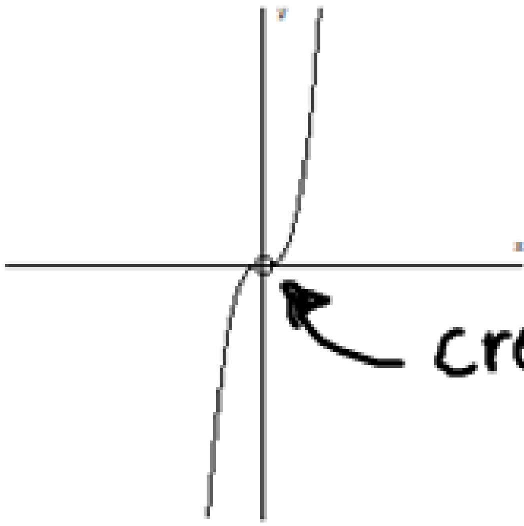
start QII
end QI

← Start low
← end low

Polynomial of degree 3: $y = ax^3 + bx^2 + cx + d$

- Cubic function
- If "a" is positive

If "a" is negative R_y

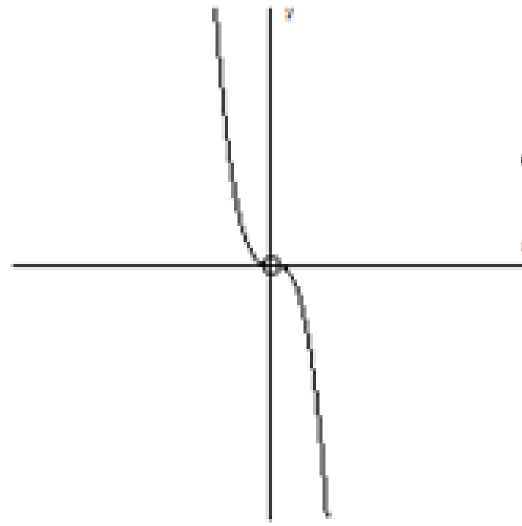


$y = x^3$

start low end high

cross

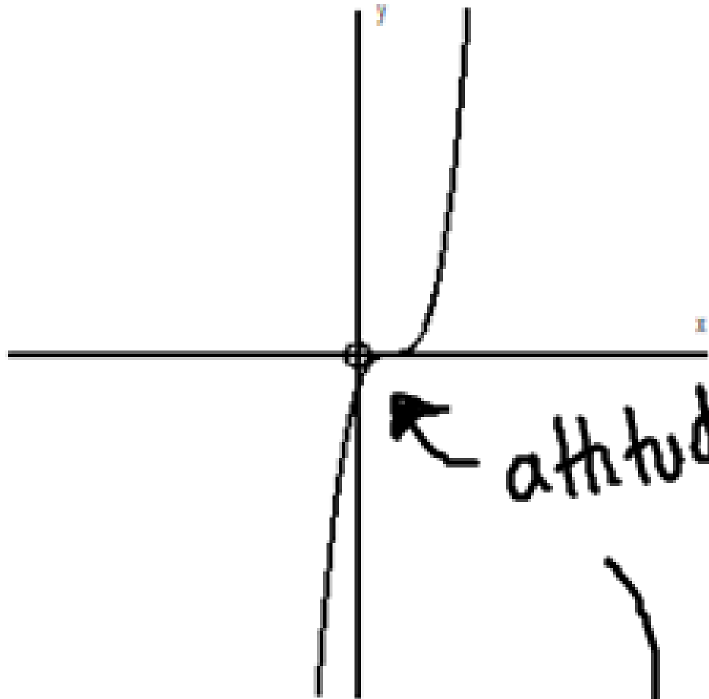
"attitude"



Start high end low

- Function will always have a x-intercept

One x-int



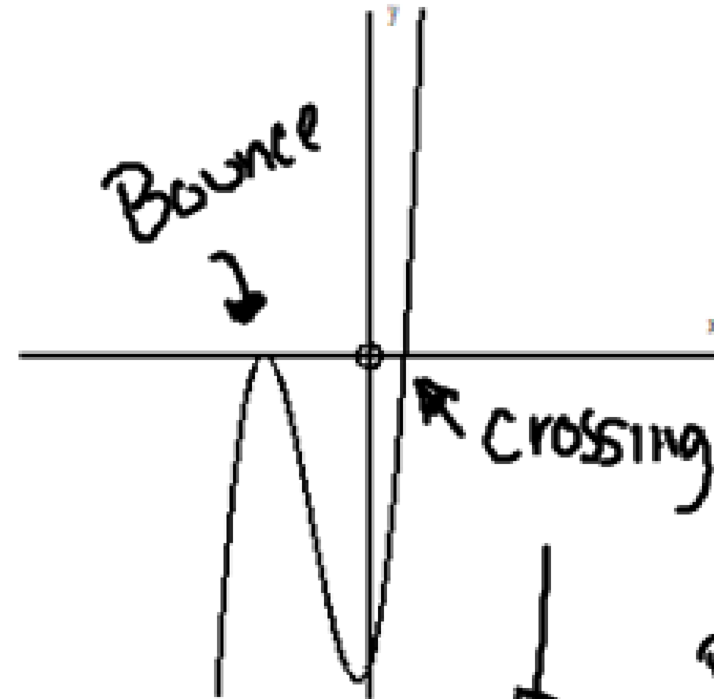
attitude

Equation: $y=(x-1)^3$

HT=1

TWO x-int

Bounce

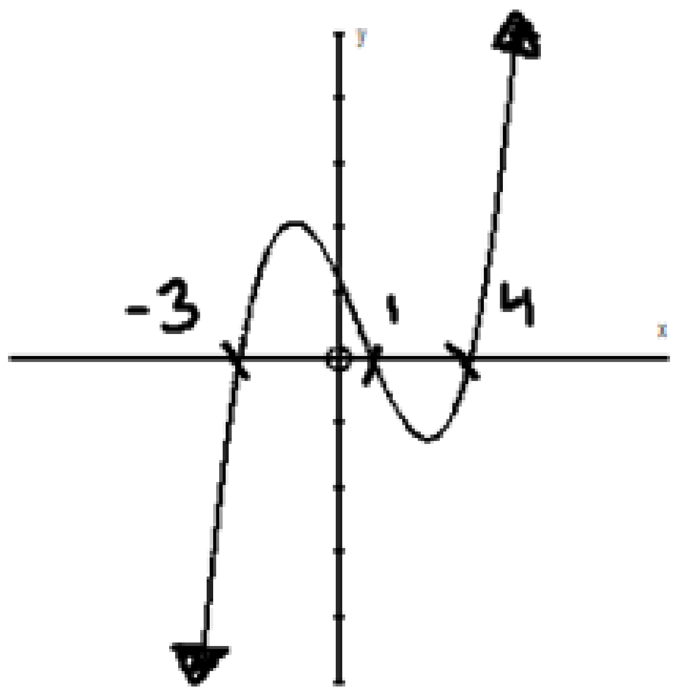


crossing

Bounce

Equation: $y=(x-1)(x+3)^2$

THREE x-int

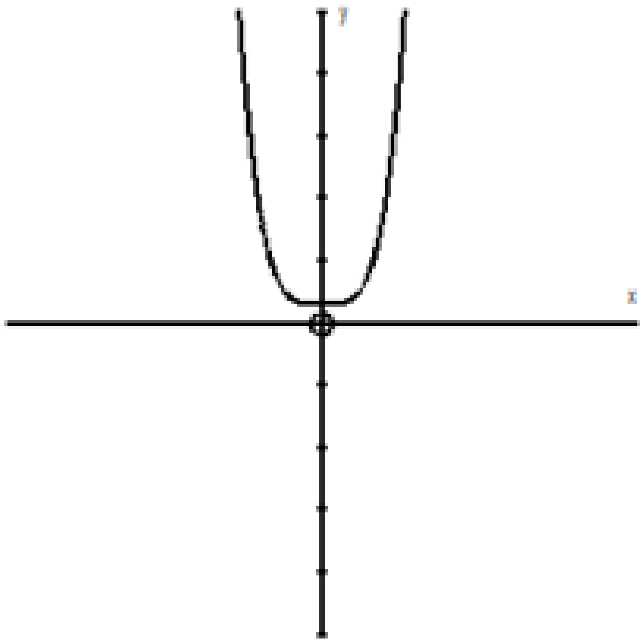


Equation: y=(x–1)(x+3)(x–4)

Polynomial of degree 4: $y = ax^4 + bx^3 + cx^2 + dx + e$

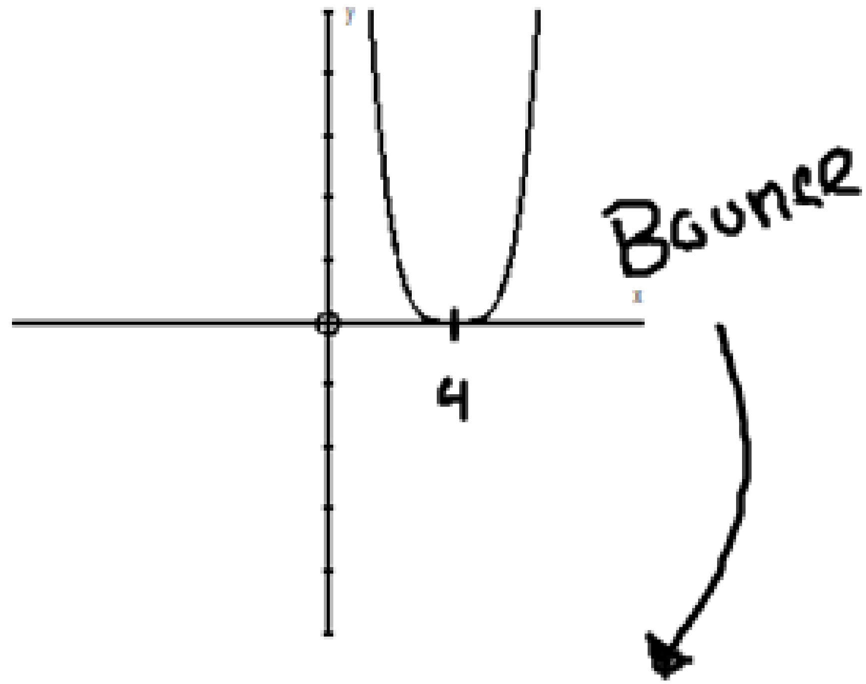
- Quartic
- If "a" is positive, graph opens up
- If "a" is negative, graph opens down

NO x-int (Complex roots)



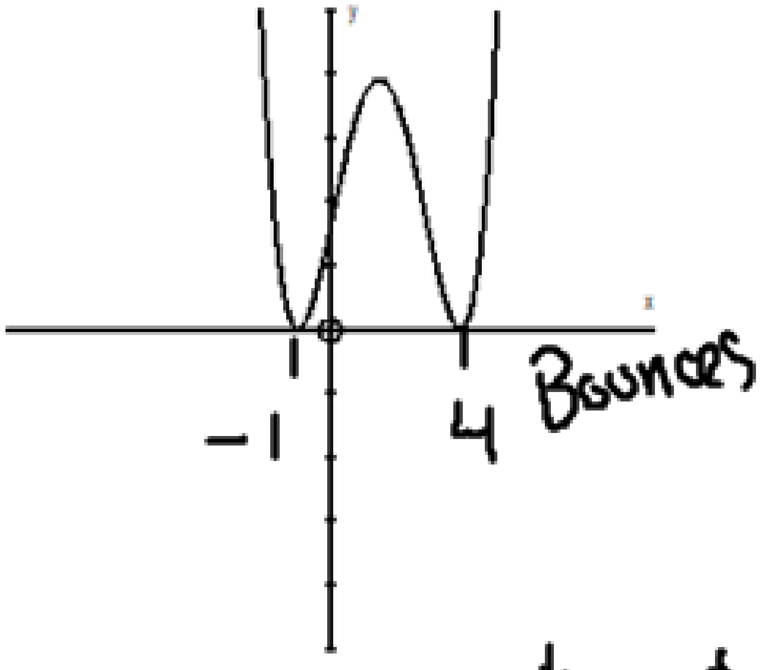
Equation: $y=(x^4+3)$

ONE x-int

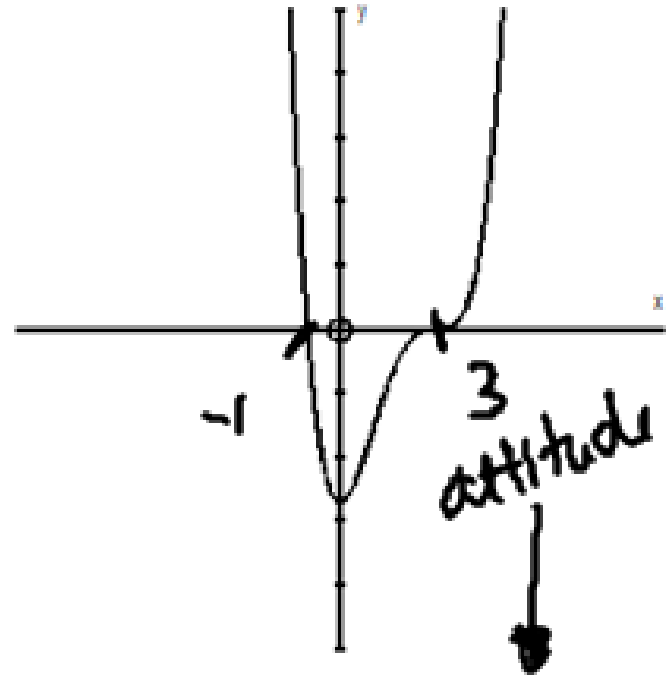


Equation: $y=(x-4)^4$

TWO x-int

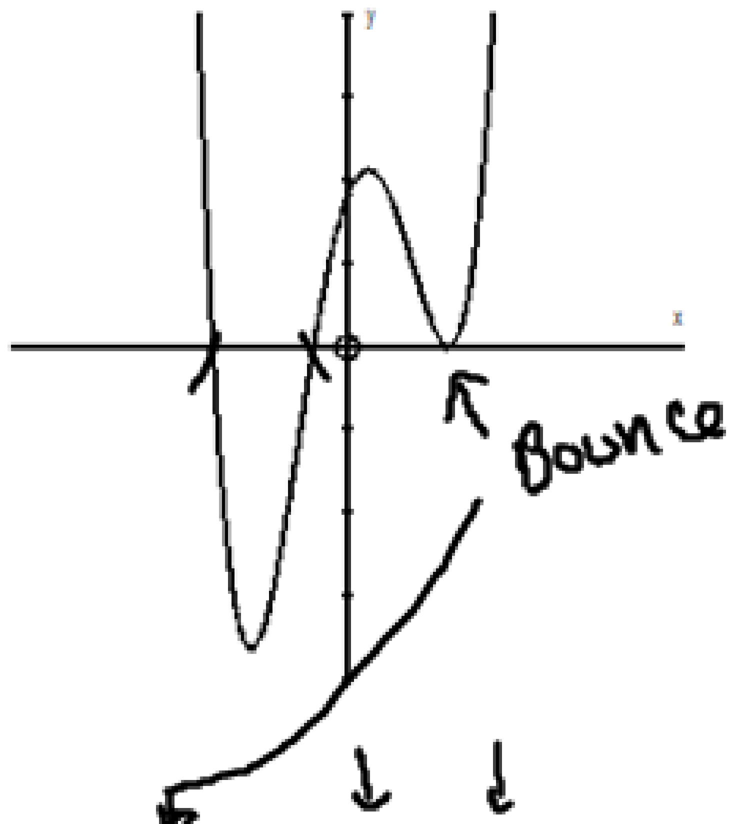


Equation: $y=(x-4)^2(x+1)^2$



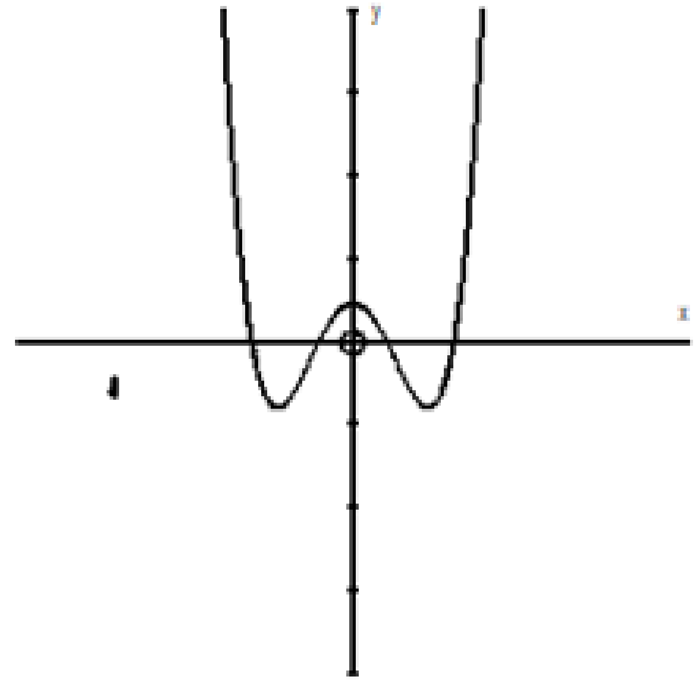
Equation: $y=(x-3)^3(x+1)$

THREE x-int

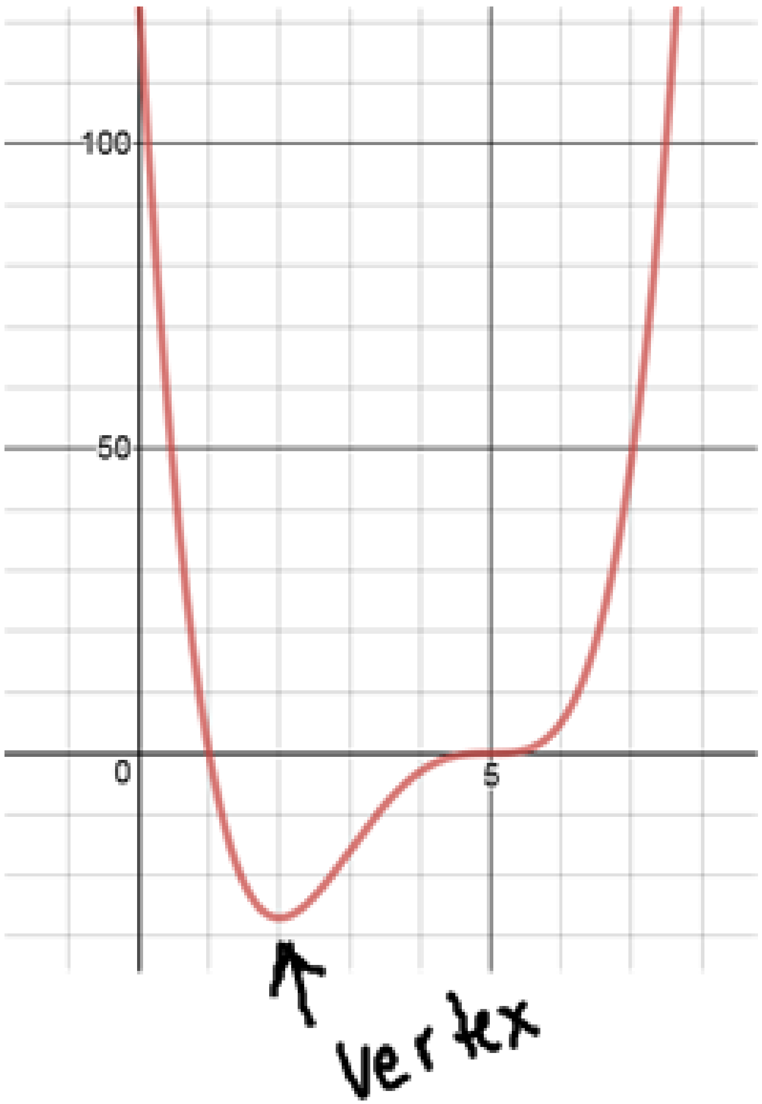


$y=(x-3)^2(x+1)(x+4)$

FOUR x-int



$y=(x-1)(x+1)(x+3)(x-3)$



End Behaviour

→ start high
end high

x-int: (1,0)
(5,0)

y-int (0,125)

#vertices 1

repeat roots: Yes at x=5
attitude

End Behaviour:
In general:

If “n” is even and:

“a” is positive

“a” is negative

If “n” is odd and

“a” is positive

“a” is negative

Example:

$y = x^3$

$y = (x-1)(x+2)(x-3)$

$y = -(x-1)(x+2)(x+2)$

HW; pg 114 #1-4