Example: (pg. 622)

$\left(\frac{72}{100} + \frac{28}{100}\right)^5$

72% of union members are in favour of a certain change to their conditions of employment. A random sample of five members is taken. Find:

(a) the probability that three members are in favour of the change in conditions.

let X = # of member in favour of change.

$X = 0, 1, 2, 3, 4, 5$

$X \sim B(5, 0.72)$ ← success (0.72), ← # trials (5)

"X is distributed as a function"

B – Binomial

$P(X=3) = 0.292626432$

```
binompdf
trials:5
p:.72
x value:3
Paste
```

```
binompdf(5,.72,
          .292626432
```

(b) the probability that at least three members are in favour of the conditions.

binomcdf

$$P(X \geq 3) = 1 - P(X \leq 2)$$

$$= 0.86235$$

```
binomcdf
trials:5
p:.72
x value:2

binomcdf(5,.72,▸
          .1376478208
1-.1376478208
          .8623521792
```

(c) the expected number of members in the sample that are in favour of the change.

$$E = n \cdot p$$

$$= (5)(0.72)$$

$$= 3.6$$

23D.3 – The Mean and Standard Deviation of a Binomial Distribution

Suppose X is a binomial random variable with parameters n and p, so $X \sim B(n, p)$.

mu μ

The mean of X is $\mu = np$.

σ

The standard deviation of X is $\sigma = \sqrt{np(1-p)}$.

The variance of X is $\sigma^2 = np(1-p)$.

Example:

A coin is tossed 5 times and X is the number of heads which occur. Find the mean, standard deviation and variance of the X-distribution.

$n = 5$ $p = \frac{1}{2}$ (H or T)

mean

$\mu = np$

$= 5(\frac{1}{2})$

$= 2.5$

Variance: 1.25

St. dev

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{2.5(0.5)}$

$= \sqrt{1.25} = 1.118$

Pg. 625

← # trials.

1 Suppose $X \sim B(6, p)$. For each of the following cases:

- **i** find the mean and standard deviation of the X-distribution
- **ii** graph the distribution using a column graph
- **iii** comment on the shape of the distribution.

b $p = 0.2$

$X \sim B(6, 0.2)$

Mean

$\mu = np$

$= 6(0.2)$

$= 1.2$

st.dev

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{1.2(1-0.2)}$

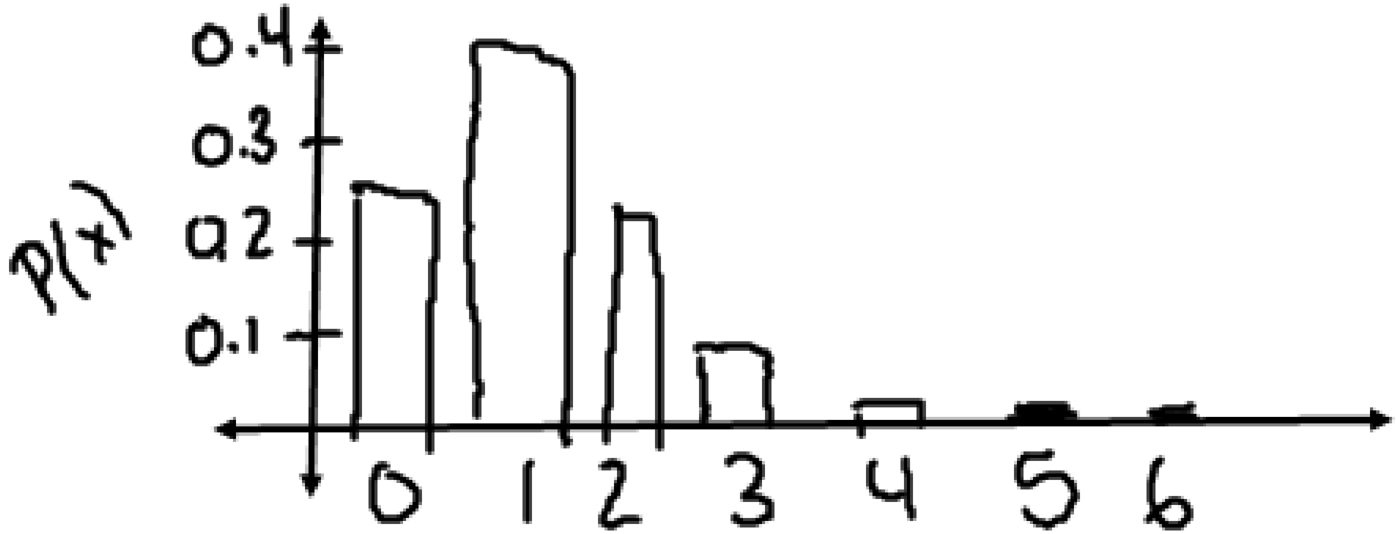
$= \sqrt{1.2(0.8)}$

$= \sqrt{0.96}$

$= 0.9798$

x	0	1	2	3	4	5	6
P(X=x)	0.262144	0.393216	0.24576	0.08192	0.01536	0.001536	0.000064

use binompdf to find these values



review ch20

The distribution is positively skewed.

Chapter 24

The normal distribution

Syllabus reference: 5.9

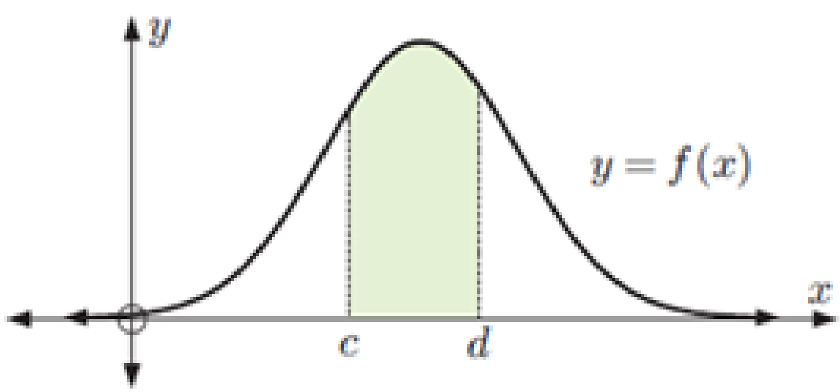
Contents:

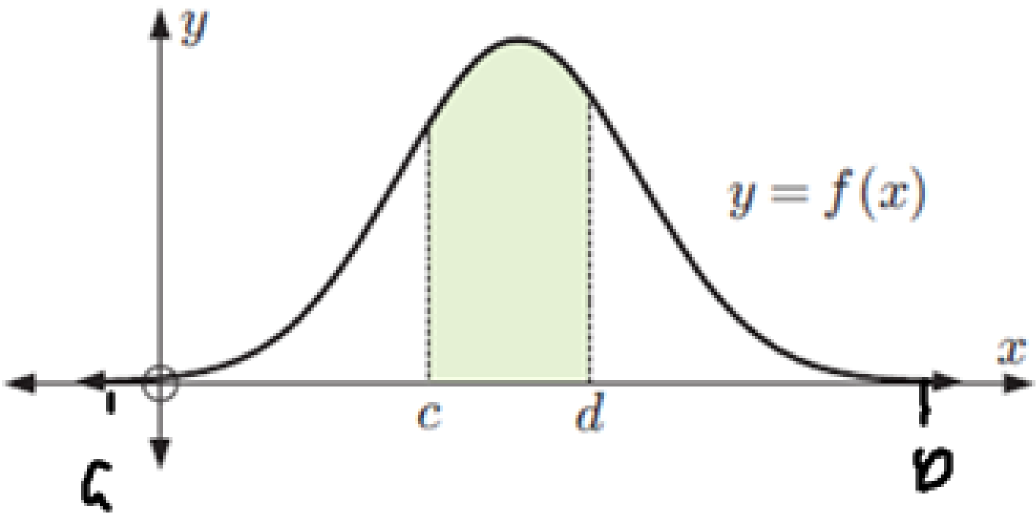
24A – The Normal Distribution

From Ch. 23: **discrete** random variables
- examined binomial probability distributions where X could take non-negative integer values, $x = 0, 1, 2, 3, \ldots$

For a **continuous** random variable X, x can be any real value.

Use a **probability density function** to define the probability distribution.
Probabilities are found be calculating areas under the probability density function curve.





For a continuous random variable X, the **probability density function** is a function $f(x)$ such that $f(x) \geqslant 0$ on its entire domain.

If the domain of the function is $a \leqslant x \leqslant b$, then $\int_a^b f(x)\,dx = 1$.

The probability that X lies in the interval $c \leqslant X \leqslant d$ is $\mathbf{P}(c \leqslant X \leqslant d) = \int_c^d f(x)\,dx$.

For a continuous variable X, the probability that X is *exactly* equal to a particular value is zero:

$$P(X = x) = 0 \text{ for all } x.$$

For example, the probability that an egg will weigh *exactly* 72.9 g is zero.

If you were to weigh an egg on scales that weigh to the nearest 0.1 g, a reading of 72.9 g means the weight lies somewhere between 72.85 g and 72.95 g. No matter how accurate your scales are, you can only ever know the weight of an egg within a range.

So, for a continuous variable we can only talk about the probability that an event lies in an **interval**.

A consequence of this is that $P(c \leqslant X \leqslant d) = P(c < X \leqslant d) = P(c \leqslant X < d) = P(c < X < d)$.

This would not be true if X was discrete.

THE NORMAL PROBABILITY DENSITY FUNCTION

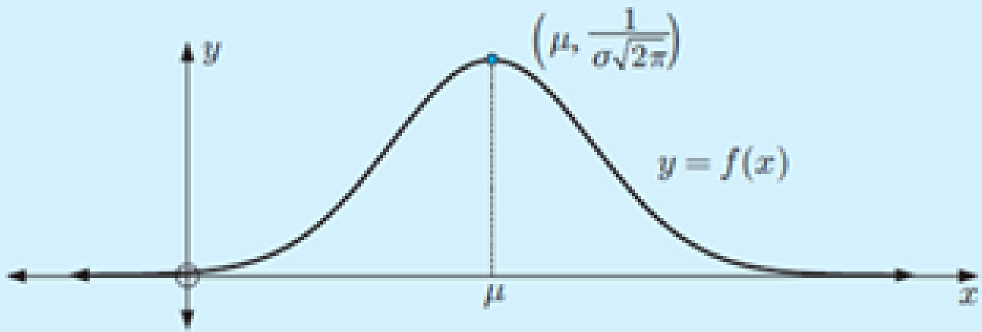
If X is **normally distributed** then its **probability density function** is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for} \quad -\infty < x < \infty$$

where μ is the mean and σ^2 is the variance of the distribution.

We write $X \sim \mathrm{N}(\mu, \sigma^2)$.

- The curve $y = f(x)$, which is called a **normal curve**, is symmetrical about the vertical line $x = \mu$.
- As $x \to \pm\infty$ the normal curve approaches its asymptote, the x-axis.
- $f(x) > 0$ for all x.
- Since the total probability must be 1, $\int_{-\infty}^{\infty} f(x)\,dx = 1$.



- More scores are distributed closer to the mean than further away. This results in the typical **bell shape**.

THE GEOMETRIC SIGNIFICANCE OF μ AND σ

Differentiating $\quad f(x)=\frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

we obtain $\quad f'(x)=\frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{x-\mu}{\sigma}\right) e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$\therefore \quad f'(x)=0$ only when $x=\mu$, and this corresponds to the point on the graph when $f(x)$ is a maximum.

Differentiating again, we obtain $\quad f''(x)=\frac{-1}{\sigma^2\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[\frac{1}{\sigma}-\frac{(x-\mu)^2}{\sigma^3}\right]$

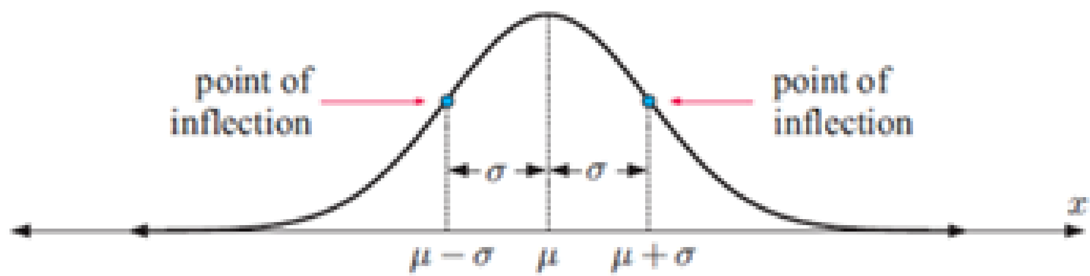
$$\therefore \quad f''(x)=0 \quad \text{when} \quad \frac{(x-\mu)^2}{\sigma^3}=\frac{1}{\sigma}$$

$$\therefore \quad (x-\mu)^2=\sigma^2$$

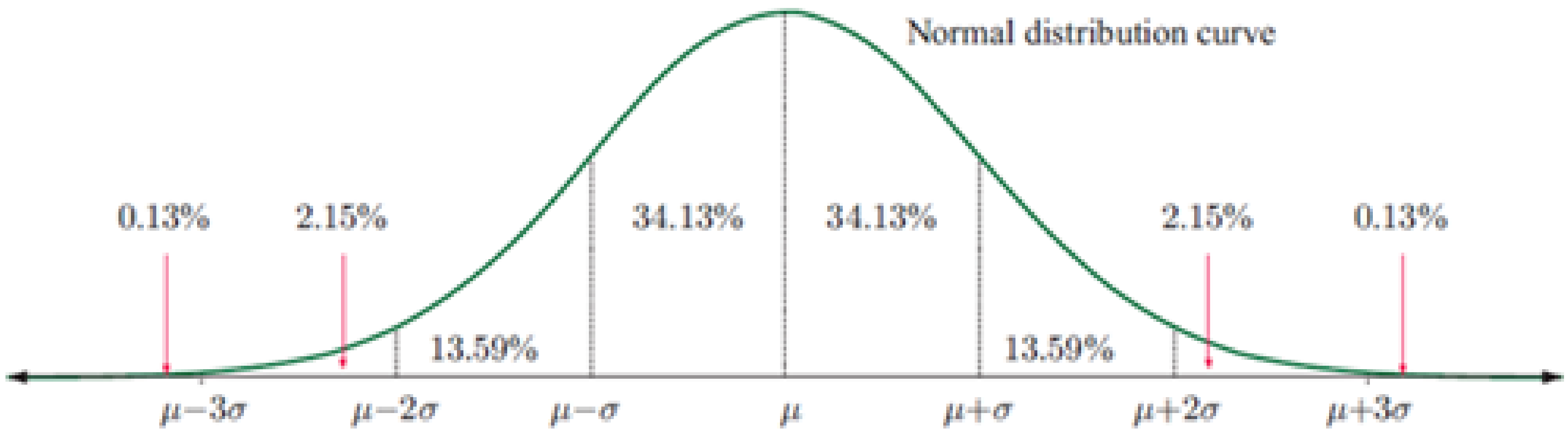
$$\therefore \quad x-\mu=\pm\sigma$$

$$\therefore \quad x=\mu\pm\sigma$$

So, the points of inflection are at $x=\mu+\sigma$ and $x=\mu-\sigma$.



For a normal distribution with mean μ and standard deviation σ, the percentage breakdown of where the random variable could lie is shown below.

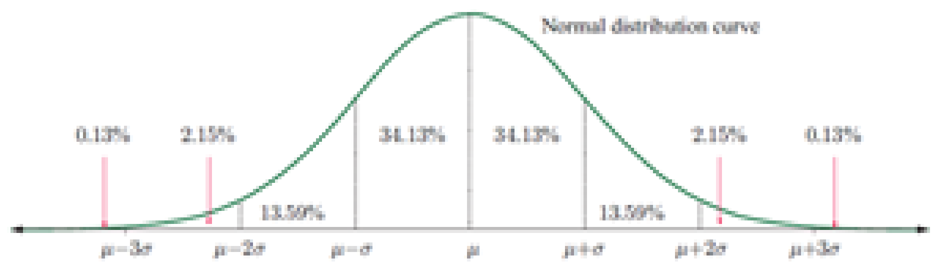


Notice that:

- $\approx 68.26\%$ of values lie between $\mu - \sigma$ and $\mu + \sigma$
- $\approx 95.44\%$ of values lie between $\mu - 2\sigma$ and $\mu + 2\sigma$
- $\approx 99.74\%$ of values lie between $\mu - 3\sigma$ and $\mu + 3\sigma$.

Example:

In an Oreo factory, the masses of cookies are normally distributed with a mean of 40 g and a standard deviation of 2 g.



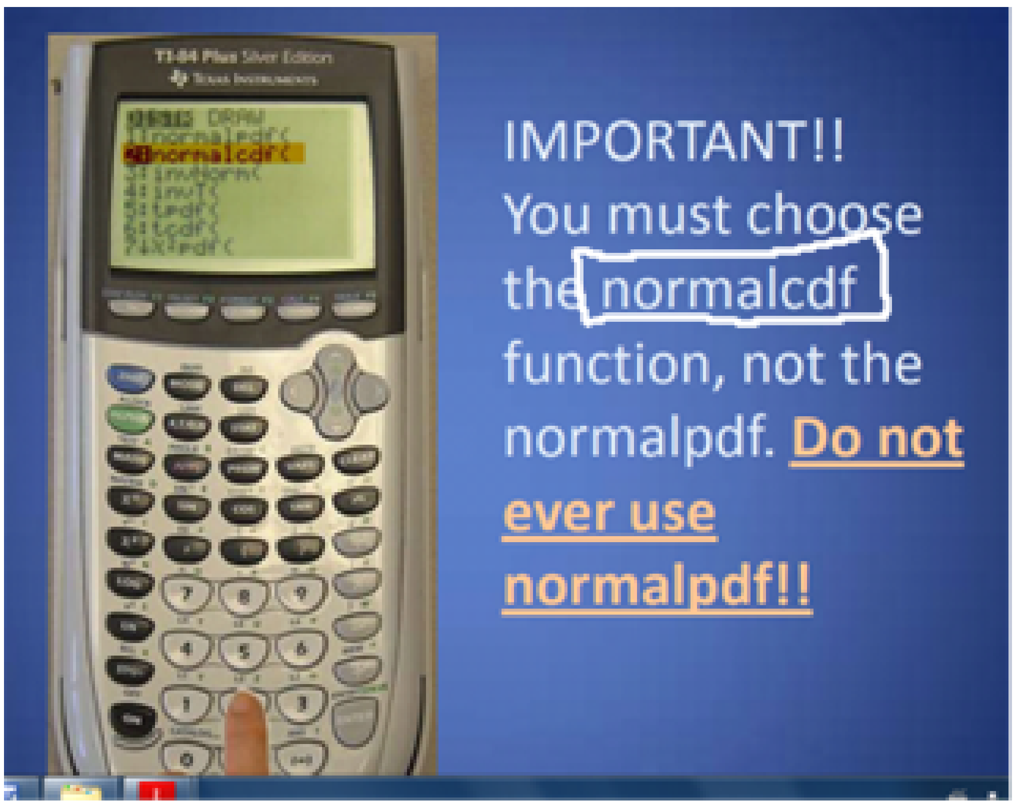
(a) Find the percentage of cookies with a mass between 36 g and 42 g.

(b) Find the probability that the mass of a cookie is between 38 g and 46 g.

(c) Find the value of k such that approximately 16% of the cookies are below k g.

24 B – Probabilities Using a Calculator

Need to use the calculator to find probabilities that are not just regions of width of σ from the mean.



Example:

Suppose $X \sim N(50, 3^2)$.

This means X is normally distributed with mean 50 and standard deviation 3.

Find:

(a) $P(40 \le X \le 65)$

(b) $P(X \ge 35)$

(c) $P(X \le 42)$