Chapter 23

# Discrete random variables

**Syllabus reference: 5.7, 5.8**

**Contents:**

# 23 A – Discrete Random Variables

A **random variable** represents in number form the possible outcomes which could occur for some random experiment.

A **discrete random variable** $X$ has a set of distinct possible values: $x_1, x_2, x_3, \ldots, x_n$.

To determine the value of a discrete random variable, need to **count**.

Example:

- Flipping a coin: the number of heads could be any **integer** value between 0 and infinity.

A **continuous random variable** $X$ could take possible values in some interval.

To determine the value of a continuous random variable, need to **measure**.

Example:

- The weight of firefighters that lie in the interval $150 < X < 300$ pounds.

**Probability Distributions**

- describes the probability that the variable will take any particular value.

The probability that the variable $X$ takes value $x$ is $P(X = x)$.

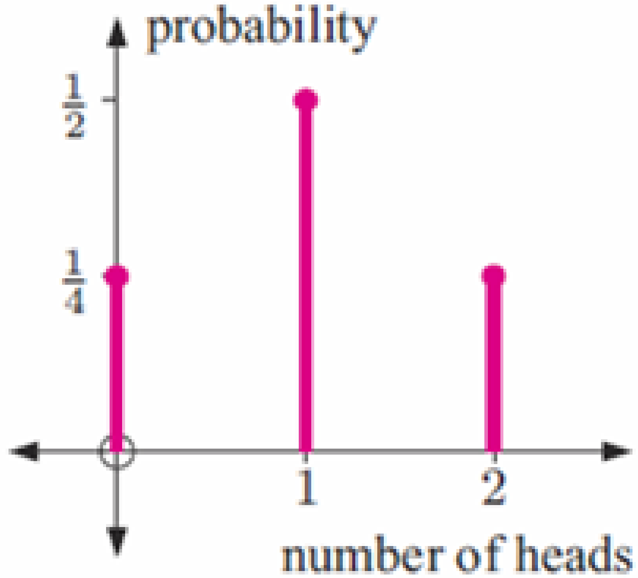
Example: Flipping two coins

HH, TT, HT, TH
2 0 1 1

$X$ is the random variable of the number of heads.
This means $X$ could be 0, 1, or 2.

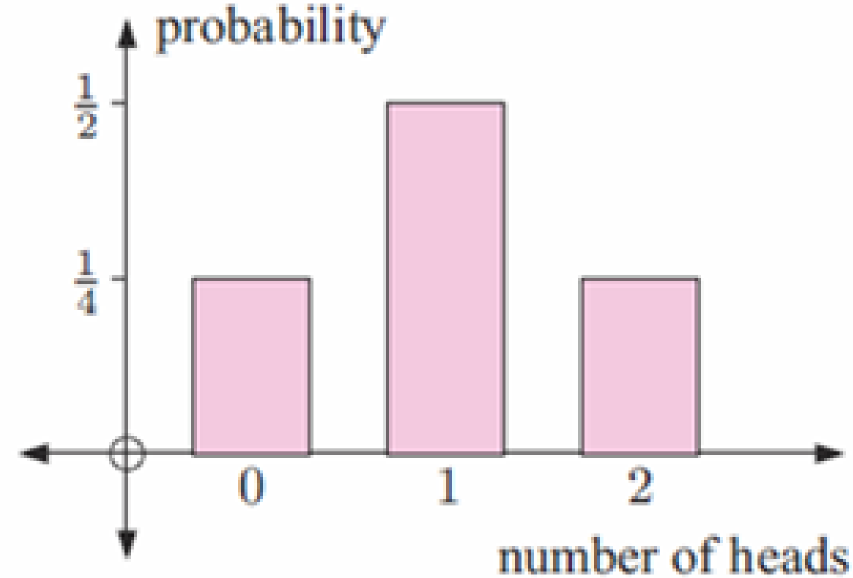
The probability distribution is:

**Spike graph**




or

**Probability column graph**



## 22B – Discrete Probability Distributions

When probabilities are assigned to events, they must satisfy the following rule:

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If $X$ is a random variable with sample space $\{x_1, x_2, x_3, ...., x_n\}$ and corresponding probabilities $\{p_1, p_2, p_3, ...., p_n\}$ so that $P(X = x_i) = p_i, \quad i = 1, ...., n,$ then:

- $0 \leqslant p_i \leqslant 1$ for all $i = 1$ to $n$
- $\sum_{i=1}^{n} p_i = p_1 + p_2 + p_3 + .... + p_n = 1$
- $\{p_1, ...., p_n\}$ describes the **probability distribution** of $X$.

The probability distribution of a discrete random variable can be given in:

- Table form
- Graphical form
- Function form, $P(x) = P(X = x)$

Careful with language!

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| *Notation* | *Statement* |
|---|---|
| $P(X = 3)$ | the probability that $X$ equals 3 |
| $P(X < 3)$ | the probability that $X$ is less than 3 |
| $P(X \leqslant 3)$ | the probability that $X$ is at most 3 |
| $P(X > 3)$ | the probability that $X$ is more than 3 |
| $P(X \geqslant 3)$ | the probability that $X$ is at least 3 |
| $P(3 < X < 7)$ | the probability that $X$ is between 3 and 7 |
| $P(3 \leqslant X \leqslant 7)$ | the probability that $X$ is at least 3 but no more than 7 |
| $P(3 < X \leqslant 7)$ | the probability that $X$ is more than 3 but no more than 7 |
| $P(3 \leqslant X < 7)$ | the probability that $X$ is at least 3 but less than 7 |

Example:

1. In a class of 100 students, 80 students passed in all subjects, 10 failed in one subject, 7 failed in two subjects and 3 failed in three subjects.

(a) What is the random variable?

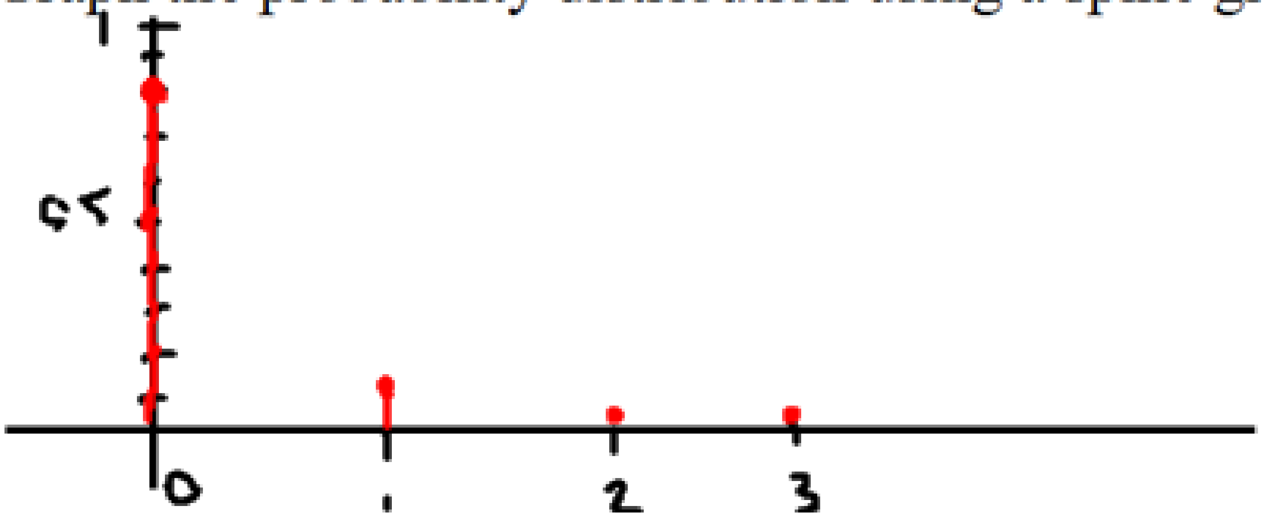
X is the number of subjects a student has failed

(b) Make a probability table for the random variable.

X = 0, 1, 2, 3,

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X=x)$ | $\frac{80}{100}$ | $\frac{10}{100}$ | $\frac{7}{100}$ | $\frac{3}{100}$ |
| | $P(X=0)=0.8$ | $P(X=1)=0.1$ | $P(X=2)=0.07$ | $P(X=3)=0.03$ |

(c) Graph the probability distribution using a spike graph.



2. Show that the following is a probability distribution function:

$$P(x) = \binom{4}{x}(0.7)^x(0.3)^{4-x}, \quad x = 0, 1, 2, 3, 4$$

$P(0) = \binom{4}{0}(0.7)^0(0.3)^4 = 0.0081$

$P(1) = \binom{4}{1}(0.7)^1(0.3)^3 = 0.0756$

$P(2) = \binom{4}{2}(0.7)^2(0.3)^2 = 0.2646$

$P(3) = \binom{4}{3}(0.7)^3(0.3)^1 = 0.4116$

$P(4) = \binom{4}{4}(0.7)^4(0.3)^0 = 0.2401$

Total = 1.0000

→ All probabilities are between 0 and 1 and $\sum_{i=1}^{4} P(x_i) = 1$

∴ $P(x)$ represents a prob. dist. fun.

Binomial coefficient

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Pascal Triangle.

row 0 → 1

row 1 → 1 1

1 2 1

$\binom{3}{0} \rightarrow {}_3C_0$ — 1 3. 3 1.

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

3. Two marbles are randomly selected without replacement from a bag containing 3 red and 4 blue marbles. Let $X$ denote the number of red marbles selected.

(a) Find the probability distribution of $X$.

(b) Illustrate the probability distribution using a spike graph.

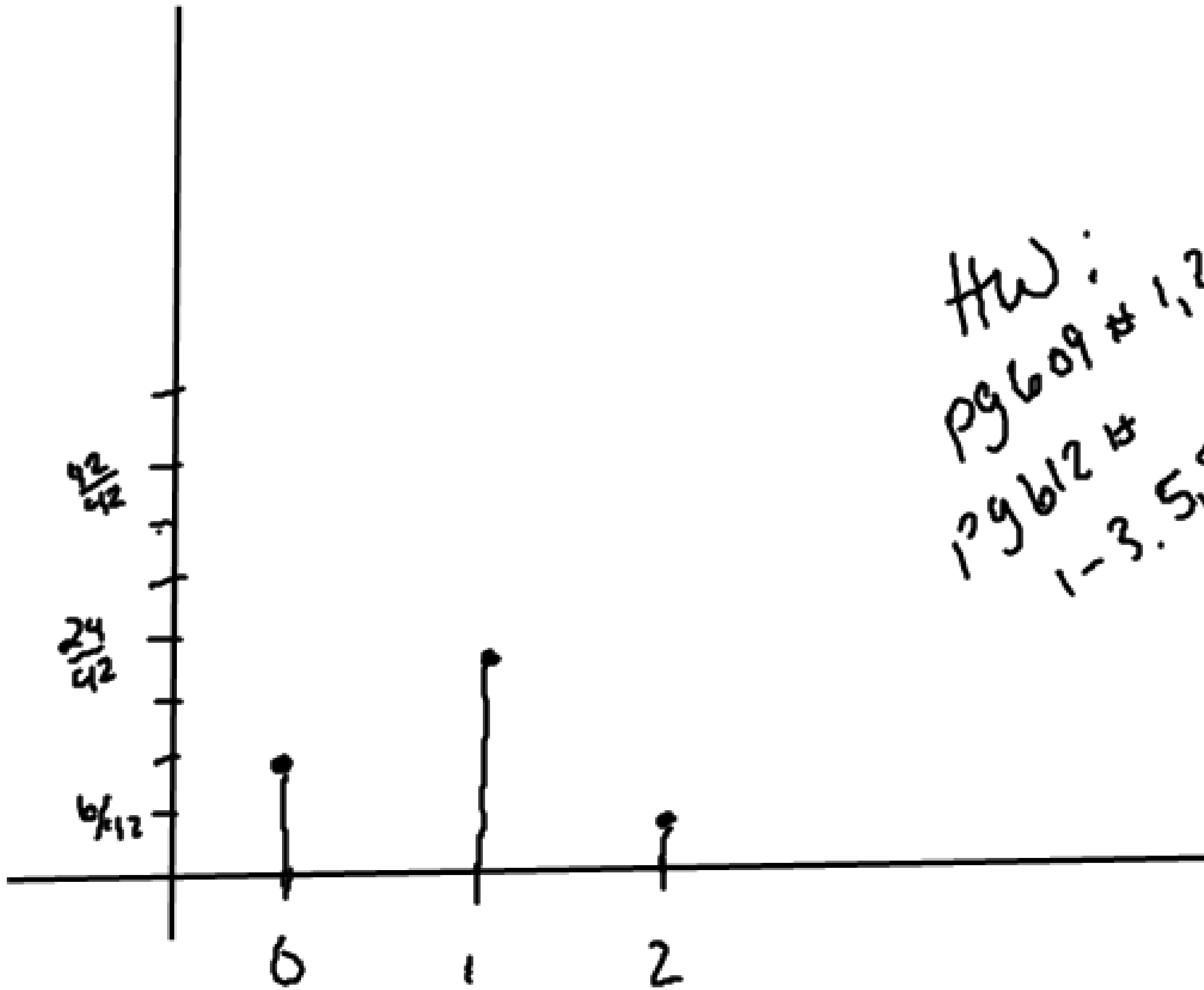
RR : $P(RR) = \left(\frac{3}{7}\right)\left(\frac{2}{6}\right) = \frac{6}{42} = P(X=2)$

RB : $P(RB) = \left(\frac{3}{7}\right)\left(\frac{4}{6}\right) = \frac{12}{42}$ } $P(X=1)$

BR : $P(BR) = \left(\frac{4}{7}\right)\left(\frac{3}{6}\right) = \frac{12}{42}$

BB : $P(BB) = \left(\frac{4}{7}\right)\left(\frac{3}{6}\right) = \frac{12}{42} = P(X=0)$

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| | $\frac{12}{42}$ | $\frac{12}{42}+\frac{12}{42}$ <br> $\frac{24}{42}$ | $\frac{6}{42}$ |




HW:
pg 609 # 1, 2, 4
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1-3, 5, 7, 10