Ch 23 C- Expectation

If there are n trials of an experiment, and an event has the probability p of occurring in each trial, then the number of times we expect the event to occur is $E = np$

Example: if a die is rolled 240 times, on how many occasions would you expect to roll a 3?

$P(3) = \frac{1}{6}$

$E = 240\left(\frac{1}{6}\right)$

$= 40$

The expected outcome for a random variable X is the mean result μ

$$E(X) = \mu = \sum_{i=1}^{n} x_i p_i = \sum_{i=1}^{n} x_i p(X = x_i)$$

7 In a random survey of her electorate, politician A discovered the residents' voting intentions in relation to herself and her two opponents B and C. The results are indicated alongside:

A	B	C
165	87	48

a Estimate the probability that a randomly chosen voter in the electorate will vote for:

i A **ii** B **iii** C.

b If there are 7500 people in the electorate, how many of these would you expect to vote for:

i A **ii** B **iii** C?

A) Total voters $= 165 + 87 + 48$
$= 300$

i) $P(A) = \frac{165}{300} \sim 0.55$

ii) $P(B) = \frac{87}{300} \sim 0.29$

iii) $P(C) = \frac{48}{300} \sim 0.16$

B)

i) $E(A) = 7500\left(\frac{165}{300}\right)$
$= 4125$

ii) $E(B) = 7500\left(\frac{87}{300}\right)$
$= 2175$

iii) $E(C) = 7500\left(\frac{48}{300}\right)$
$= 1200$

13 A pair of dice is rolled and the random variable M is the larger of the two numbers that are shown uppermost, or the value of a single die if a double is thrown.

a In table form, obtain the probability distribution of M.

b Find the mean of the M-distribution.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

m	1	2	3	4	5	6
$P(M=m)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$$\mu = \sum_{i=1}^{6} m_i (P_i)$$

$$\mu = (1)\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

$$\mu = \frac{1+6+15+28+45+66}{36} = \frac{161}{36} \sim 4.47$$

Ch 23D – The Binomial Distribution

If an experiment for which there are two possible results, **success** or **failure**, is repeated in an independent number of trials, it is called a binomial experiment.

The probability of a success, p, and the probability of a failure, $1 - p$, must be constant for all trials.

X is the random variable that is the total number of successes in n trials.

Example:

Complete a probability distribution table for tossing a fair coin 3 times (looking at the probability for the number of heads).

X=#heads	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

HHH TTT
HHT TTH
HTH THT
HTT THH

$\underline{2} \cdot \underline{2} \cdot \underline{2} = 8$

Now complete a probability table for tossing a fair coin 4 times.

_ _ _ _

X=#heads	0	1	2	3	4
P(X=x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

HHHH TTTT
HHHT TTTH
HHTH TTHT
HTHH THTT
HHTT TTHH
HTTH THHT
HTHT THTH
HTTT THHH

Pascal's Triangle

Pascals Triangle

row 0 → 1 $\quad {}_0C_0$

row 1 → 1 1 $\quad {}_1C_0, {}_1C_1$

row 2 → 1 2 1 $\quad {}_2C_0, {}_2C_1, {}_2C_2$

row 3 → 1 3 3 1

row 4 → 1 4 6 4 1 $\quad {}_4C_2$

row 5 → 1 5 10 10 5 1 $= 2^5$

binomial coefficient

$\binom{n}{r}$ calc: ${}_nC_r$

Example:

You are playing a game where you have a 70% chance of winning.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
→ 1 5 10 10 5 1

(a) Expand $\left(\frac{7}{10}+\frac{3}{10}\right)^5$

$$= (1)\left(\frac{7}{10}\right)^5 + (5)\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) + 10\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 + 10\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + 5\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)^4 + (1)\left(\frac{3}{10}\right)^5$$

(b) If 5 games are played, what is the probability of winning:

(i) exactly 3 games (ii) less than 3 games

Prob of winning each game is $\frac{7}{10}$

let X be the number of Games won

$$= (1)\left(\frac{7}{10}\right)^5 + (5)\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) + 10\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2 + 10\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + 5\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)^4 + (1)\left(\frac{3}{10}\right)^5$$

$P(X=5)$	$P(X=4)$	$P(X=3)$	$P(X=2)$	$P(X=1)$	$P(X=0)$
5 wins	4 wins	3 wins	2 wins	1 win	0 win
0 loss	1 loss	2 loss	3 loss	4 loss	5 loss

i) $P(X=3) = 10\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$

$= 0.3087$

ii) $P(X=2) + P(X=1) + P(X=0)$

$= 10\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3 + 5\left(\frac{7}{10}\right)\left(\frac{3}{10}\right)^4 + (1)\left(\frac{3}{10}\right)^5$

$= 0.1323 + 0.02835 + 0.00243$

$= 0.16308$

Pg. 421

THE BINOMIAL PROBABILITY DISTRIBUTION FUNCTION

Consider a binomial experiment for which p is the probability of a *success* and $1-p$ is the probability of a *failure*.

If there are n independent trials then the probability that there are r *successes* and $n-r$ *failures* is $P(X=r) = \binom{n}{r} p^r(1-p)^{n-r}$ where $r = 0, 1, 2, 3, 4, \ldots., n$.

$P(X=r)$ is the **binomial probability distribution function.**

The **expected** or **mean** outcome of the experiment is $\mu = E(X) = np$.

If X is the random variable of a binomial experiment with parameters n and p, then we write $X \sim B(n, p)$ where $\sim$ reads "*is distributed as*".

Graphing calculator can be used to calculate binomial probabilities:

To find the probability $P(X=r)$ that the variable takes the value r, use the binomial probability distribution function.

To find the probability that the variable takes a range of values, such as $P(X \leq r)$, use the binomial cumulative distribution function.

Example: (pg. 622)

72% of union members are in favour of a certain change to their conditions of employment. A random sample of five members is taken. Find:

(a) the probability that three members are in favour of the change in conditions.

(b) the probability that at least three members are in favour of the conditions.

(c) the expected number of members in the sample that are in favour of the change.

HW pg 616 ch23C
even Q #1-14.

ch 23D.1 pg 621
#1-5

$\boxed{2nd}\ \boxed{vars}\ \boxed{A}$

$P(X = x)$

$\boxed{2nd}\ \boxed{vars}\ \boxed{B}$

$P(X \leq x)$