22I – Laws of Probability

The Addition Law:

For two events, A and B: "or"

union

intersect "and"

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

overlap

If A and B are mutually exclusive events, then $P(A \cap B) = 0$ so the addition law becomes $P(A \cup B) = P(A) + P(B)$.

Example:

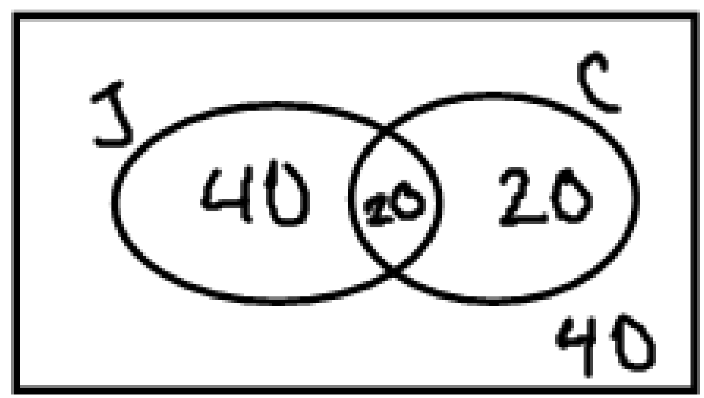
120 students were polled: 60 had part time jobs, 40 had clubs outside of school, 20 had both a job and a club. What is the probability that a student selected has a job or is involved in a club?

$P(J) = \frac{60}{120}$

$P(C) = \frac{40}{120}$

$P(J \cap C) = \frac{20}{120}$

$$P(J \cup C) = P(J) + P(C) - P(J \cap C)$$
$$= \frac{60}{120} + \frac{40}{120} - \frac{20}{120}$$
$$= \frac{80}{120}$$



Example: Suppose we select a card at random from a pack of 52 cards.

(a) What is the probability that the card is an ace or a seven?

$P(A) = \frac{4}{52}$ $P(7) = \frac{4}{52}$ $P(A \cap 7) = 0$

$$P(A \cup 7) = P(A) + P(7)$$
$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

(b) What is the probability that the card is a heart or a seven?

$P(\heartsuit) = \frac{13}{52}$ $P(7) = \frac{4}{52}$ $P(\heartsuit \cap 7) = \frac{1}{52}$

$$P(\heartsuit \cup 7) = P(\heartsuit) + P(7) - P(\heartsuit \cap 7)$$
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$$

Conditional Probability:

Given two events, A and B, the conditional probability of A given B is the probability that A occurs given that B has already occurred.

A given B

intersect "and"

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Rearranging the equation gives:

$$P(A \cap B) = P(A \mid B)P(B)$$

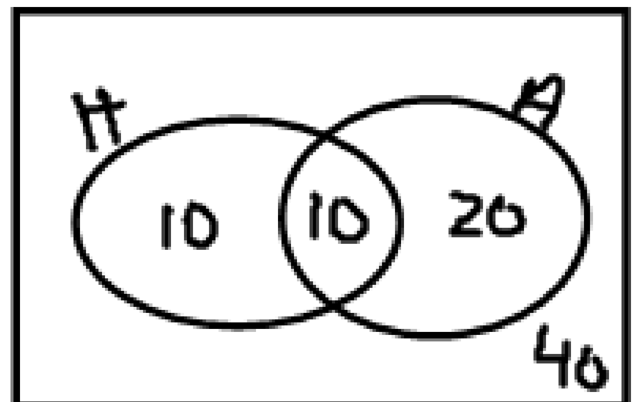
$$P(A \cap B) = P(B \mid A)P(A)$$

Example:

1. A class of 80 students has 20 honour students and 30 school athletes. Ten students are both honours and school athletes. If a student is selected at random, what is the probability that they are:

$P(H) = \frac{20}{80}$

$P(A) = \frac{30}{80}$



(a) both an honour student and a school athlete

$P(H \cap A) = \frac{10}{80}$

(b) an honour student or a school athlete

$$P(H \cup A) = P(H) + P(A) - P(H \cap A)$$
$$= \frac{20}{80} + \frac{30}{80} - \frac{10}{80} = \frac{40}{80}$$

(c) a school athlete given that they are an honour student?

$$P(A|H) = \frac{P(A \cap H)}{P(H)} = \frac{\frac{10}{80}}{\frac{20}{80}} = \frac{10}{20}$$

Pg. 601

12 On any day, the probability that a boy eats his prepared lunch is 0.5. The probability that his sister eats her lunch is 0.6. The probability that the girl eats her lunch given that the boy eats his is 0.9. Determine the probability that:

a both eat their lunch

b the boy eats his lunch given that the girl eats hers

c at least one of them eats their lunch.

$P(G) = 0.6$

$P(B) = 0.5$

$P(G|B) = 0.9$

A) $P(G \cap B) = P(G|B)P(B)$

$= (0.9)(0.5)$

$= 0.45$

b) $P(B|G) = \frac{P(B \cap G)}{P(G)}$

$= \frac{0.45}{0.6}$

$= 0.75$

c) $P(G \cup B) = P(G) + P(B) - P(B \cap G)$

$= 0.6 + 0.5 - 0.45$

$= 0.65$

22J – Independent Events

A and *B* are independent events if the occurrence of each one does not affect the probability that the other occurs.

This means:

complement
← not B

$$P(A \mid B) = P(A \mid B') = P(A)$$

Since $P(A \cap B) = P(A \mid B)P(B)$, then

and

$$P(A \cap B) = P(A)P(B)$$

Examples:

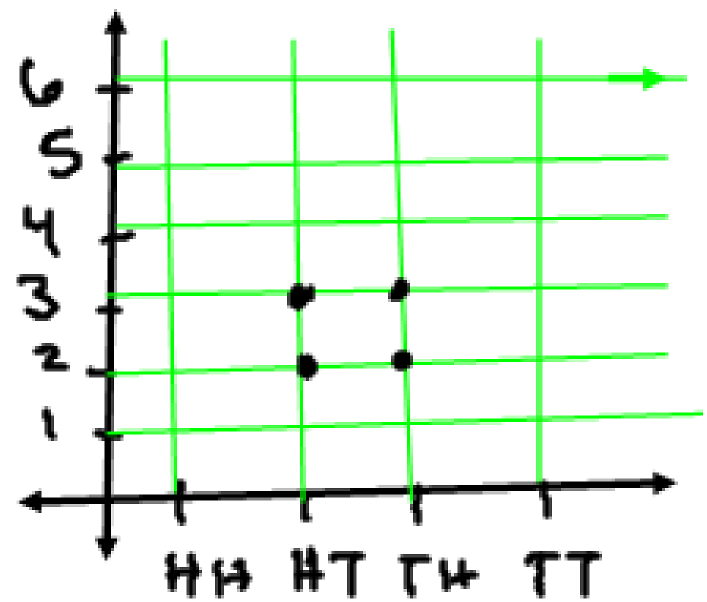
1. Suppose two coins are tossed and a die is rolled. Event A is the event of getting a head and a tail, and event B is the event of getting a 2 or 3. Show that A and B are independent events.

event A: TT, HT, TH, HH

$P(A) = \frac{2}{4}$

event B: 1, 2, 3, 4, 5, 6

$P(B) = \frac{2}{6}$



$P(A) \cdot P(B) = \left(\frac{2}{4}\right)\left(\frac{2}{6}\right) = \frac{4}{24} = \frac{1}{6}$

$P(A \cap B) = \frac{4}{24}$

Same ... independant

2. Suppose $P(A)=\frac{2}{3}$, and $P(B)=\frac{1}{4}$. Find $P(A \cup B)$ if:

or

(a) A and B are mutually exclusive

no overlap

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{3} + \frac{1}{4} - 0$$

$$= \frac{8+3}{12} = \frac{11}{12}$$

(b) A and B are independent.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A) \cdot P(B)$

$$= \frac{2}{3} + \frac{1}{4} - \left(\frac{2}{3}\right)\left(\frac{1}{4}\right)$$

$$= \frac{9}{12}$$

Hw pg 597

#1-3, 7, 8, 10

pg 602 #

1-3, 6-8

Pg. 603

8 Suppose $P(C) = \frac{9}{20}$, $P(C \mid D') = \frac{3}{7}$, and $P(C \mid D) = \frac{6}{13}$.

a Find: i $P(D)$ ii $P(C' \cup D')$

b Are C and D independent events? Give a reason for your answer.