

## 22C – Theoretical Probability

### Theoretical Probability:

The likelihood of a successful event (i.e. expected results). For example, finding the probability of rolling a 6 on a die.

$$P(A) = \frac{\text{number of members of the event } A}{\text{total number of possible outcomes}}$$

$$P(A) = \frac{n(A)}{n(U)}$$

↖ universe

For example, the probability of flipping heads is  $\frac{1}{2}$

If two coins are tossed, what is the probability that at least 1 head turns up?

sample space:  $HH, TT, HT, TH$

$$P(A) = \frac{3}{4}$$

## Complement: $P(A')$

The complement of an event  $x$  is the event of not  $x$ . For example, the complement of having blue eyes is **not** having blue eyes.

$$P(\bar{A})$$

$$P(A) + P(A') = 1$$

If two coins are tossed, what is the probability of NOT getting 1 head?

$$\begin{aligned} P(A') &= 1 - P(A) \\ &= 1 - \frac{3}{4} \\ &= \frac{1}{4} \end{aligned}$$

## Examples:

Suppose you drew one card from a well-shuffled deck of 52 playing cards. What is the probability of each event?

(a) Drawing a king ← 4 Kings

$$P(\text{king}) = \frac{4}{52}$$

(b) Drawing a red ace ← 2 red Aces ♡ ♦

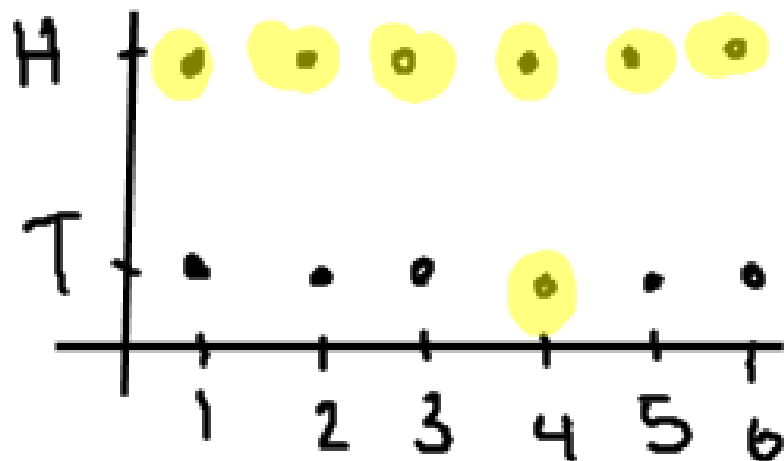
$$P(\text{RA}) = \frac{2}{52}$$

(c) Not drawing a red ace

$$P(\text{RA}') = 1 - \frac{2}{52} = \frac{50}{52}$$

Example:

Use a 2-dimensional grid to illustrate the sample space for tossing a coin and rolling a die.



T<sub>1</sub>, T<sub>2</sub>, T<sub>3</sub>, ...

12 possible outcomes

Determine the probability of:

(a) rolling a 4. =  $\frac{2}{12}$

(b) rolling a four and getting a head. =  $\frac{1}{12}$

(c) rolling a four or getting a head. =  $\frac{7}{12}$

## **22D – Tables of Outcomes**

Tables of outcomes are tables which compare two categorical variables (usually from a survey).

A sample of students were surveyed about their preference in music and ice cream:

|               | Chocolate | Vanilla | Other | <u>Total</u> |
|---------------|-----------|---------|-------|--------------|
| Rap           | 5         | 10      | 3     | 18           |
| Classical     | 12        | 3       | 3     | 18           |
| Rock          | 39        | 10      | 15    | 64           |
| Other         | 4         | 7       | 9     | 20           |
| <u>Total:</u> | 60        | 30      | 30    | 120          |

Estimate the probability that the next randomly selected person:

A) <sup>prefers</sup> likes chocolate ice cream.  $\frac{60}{120}$

B) does not like vanilla ice cream and prefers rock music.  $\frac{39+15}{120}$

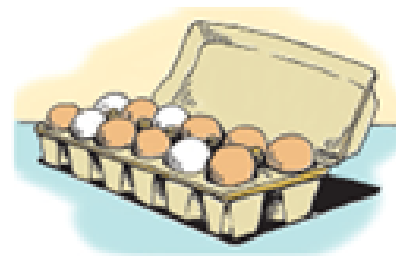
C) Prefers Rap music given that they like other flavors of ice cream.  $\frac{3}{120}$

## EXERCISE 22C.1

- 1 A marble is randomly selected from a box containing 5 green, 3 red, and 7 blue marbles. Determine the probability that the marble is:
- a red
  - b green
  - c blue
  - d not red
  - e neither green nor blue
  - f green or red.

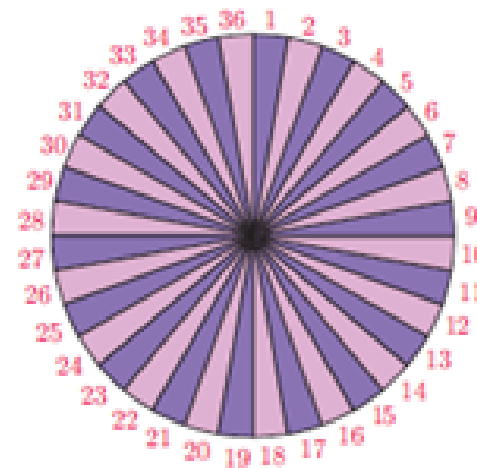
- 2 A carton of a dozen eggs contains eight brown eggs. The rest are white.

- a How many white eggs are there in the carton?
- b Find the probability that an egg selected at random is:
  - i brown
  - ii white.



- 3 A dart board has 36 sectors labelled 1 to 36. Determine the probability that a dart thrown at the centre of the board will hit a sector labelled with:

- a a multiple of 4
- b a number between 6 and 9 inclusive
- c a number greater than 20
- d 9
- e a multiple of 13
- f an odd number that is a multiple of 3
- g a multiple of both 4 and 6
- h a multiple of 4 or 6, or both.



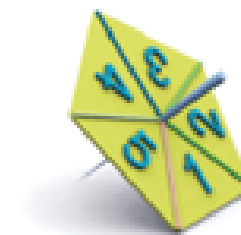
- 4 What is the probability that a randomly chosen person has his or her next birthday:
- a on a Tuesday
  - b on a weekend
  - c in July
  - d in January or February?
- 5
- a List the six different orders in which Antti, Kai, and Neda may sit in a row.
  - b If the three of them sit randomly in a row, determine the probability that:
    - i Antti sits in the middle
    - ii Antti sits at the left end
    - iii Antti does not sit at the right end
    - iv Kai and Neda are seated together.



- 6 a List the 8 possible 3-child families according to the gender of the children. For example, GGB means “the first is a girl, the second is a girl, the third is a boy”.
- b Assuming that each of these is equally likely to occur, determine the probability that a randomly selected 3-child family consists of:
- i all boys
  - ii all girls
  - iii boy then girl then girl
  - iv two girls and a boy
  - v a girl for the eldest
  - vi at least one boy.
- 7 a List, in systematic order, the 24 different orders in which four people A, B, C, and D may sit in a row.
- b Determine the probability that when the four people sit at random in a row:
- i A sits on one of the end seats
  - ii B sits on one of the two middle seats
  - iii A and B are seated together
  - iv A, B, and C are seated together, not necessarily in that order.

## EXERCISE 22C.2

- 1 Draw the grid of the sample space when a 5-cent and a 10-cent coin are tossed simultaneously. Hence determine the probability of getting:
- a two heads
  - b two tails
  - c exactly one head
  - d at least one head.
- 2 A coin and a pentagonal spinner with sectors 1, 2, 3, 4, and 5 are tossed and spun respectively.
- a Draw a grid to illustrate the sample space of possible outcomes.
  - b How many outcomes are possible?



## EXERCISE 22D

- 1 A sample of adults in a suburb were surveyed about their current employment status and their level of education. The results are summarised in the table below.

|                           | Employed | Unemployed |
|---------------------------|----------|------------|
| Attended university       | 225      | 34         |
| Did not attend university | 197      | 81         |

Estimate the probability that the next randomly chosen adult:

- attended university
  - did not attend university and is currently employed
  - is unemployed
  - is employed, given that they attended university
  - attended university, given that they are unemployed.
- 2 The types of ticket used to gain access to a basketball match were recorded as people entered the stadium. The results are shown alongside.
- |                            | Adult | Child |
|----------------------------|-------|-------|
| Season ticket holder       | 1824  | 779   |
| Not a season ticket holder | 3247  | 1660  |
- What was the total attendance for the match?
  - One person is randomly selected to sit on the home team's bench. Find the probability that the person selected:
    - is a child
    - is not a season ticket holder
    - is an adult season ticket holder.

- 3 A small hotel in London has kept a record of all the room bookings made for the year, categorised by season and booking type. Find the probability that a randomly selected booking was:

|                 | Single | Double | Family |
|-----------------|--------|--------|--------|
| Peak season     | 125    | 220    | 98     |
| Off-peak season | 248    | 192    | 152    |

- in the peak season
- a single room in the off-peak season
- a single or a double room
- a family room, given that it was in the off-peak season
- in the peak season, given that it was not a single room.

**c** Use your grid to determine the chance of getting:

- i** a tail and a 3
- ii** a head and an even number
- iii** an odd number
- iv** a head or a 5.

“A head or a 5” means “a head or a 5, or both”.



**3** A pair of dice is rolled. The 36 different possible results are illustrated in the 2-dimensional grid.

Use the grid to determine the probability of getting:

- a** two 3s
- b** a 5 and a 6
- c** a 5 or a 6 (or both)
- d** at least one 6
- e** exactly one 6
- f** no sixes
- g** a sum of 7
- h** a sum greater than 8
- i** a sum of 7 or 11
- j** a sum of no more than 8.

