

Chapter

18

Integration

Syllabus reference: 6.4, 6.5

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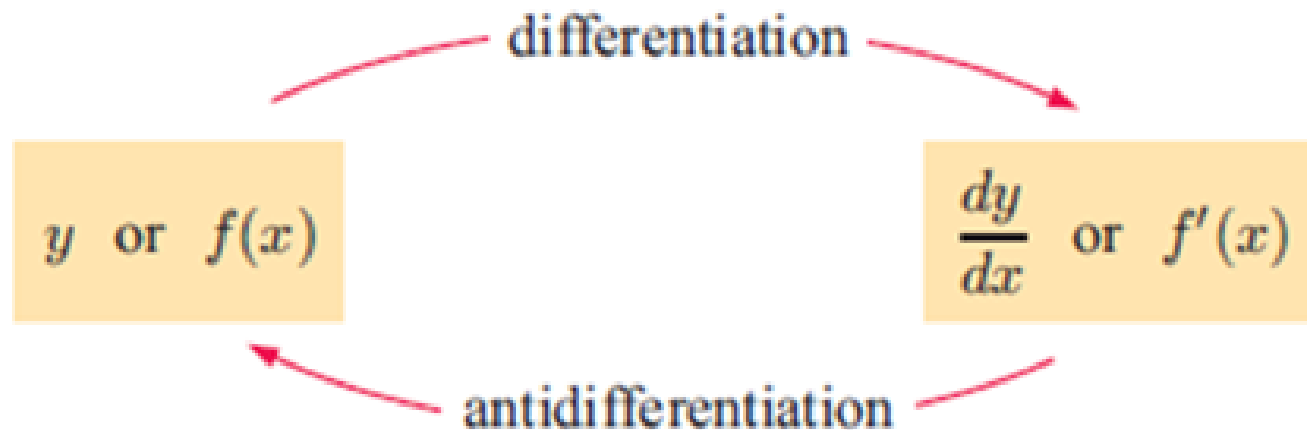
18B – Antidifferentiation

Integral Calculus involves antidifferentiation – which is the reverse process of differentiation.

Applications of integral calculus:

- Finding areas of shapes with curved boundaries
- Finding volumes of revolution
- Finding distances travelled from velocity functions
- Solving problems in economics, biology, and statistics
- Solving differential equations

Antidifferentiation is the process of finding y from $\frac{dy}{dx}$, or $f(x)$ from $f'(x)$.



Example:

Give three possible functions where $\frac{dy}{dx} = x^2$.

$$y = \frac{1}{3}x^3 + 1 \rightarrow y' = \frac{1}{3}(3x^2) \\ = x^2$$

$$y = \frac{1}{3}x^3 - \frac{15}{7}$$

$$y = \frac{1}{3}x^3 - \frac{11}{2}$$

$\frac{1}{3}x^3$ is the antiderivative of x^2 . It is the simplest function which, when differentiated, gives x^2 .

If $F(x)$ is a function where $F'(x) = f(x)$ we say that:

- the derivative of $F(x)$ is $f(x)$ and
- the antiderivative of $f(x)$ is $F(x)$.

Example: Find the antiderivative of:

$$\text{A) } x^4 \quad y' = x^4$$

$$y = \frac{1}{5} x^5$$

$$y' = x^n$$

$$y = \frac{1}{n+1} x^{n+1}$$

$$\text{B) } e^{2x}$$

$$y' = e^{2x}$$

$$y = \frac{1}{2} e^{2x}$$

$$\left\{ \begin{array}{l} \text{if} \\ y = e^{2x} \\ y' = 2e^{2x} \end{array} \right.$$

Chain Rule

$$\text{C) } x^{-\frac{1}{3}}$$

$$y' = x^{-\frac{1}{3}}$$

$$y = \frac{3}{2} x^{\frac{2}{3}}$$

$$-\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

EXERCISE 18B

1 a Find the antiderivative of:

i x ii x^2 iii x^5 iv x^{-2} v x^{-4} vi $x^{\frac{1}{3}}$ vii $x^{-\frac{1}{2}}$

b Predict a general rule for the antiderivative of x^n , for $n \neq -1$.

2 a Find the antiderivative of:

i e^{2x} ii e^{5x} iii $e^{\frac{1}{2}x}$ iv $e^{0.01x}$ v $e^{\pi x}$ vi $e^{\frac{x}{3}}$

b Predict a general rule for the antiderivative of e^{kx} where k is a constant.

3 Find the antiderivative of:

- a $6x^2 + 4x$ by first differentiating $x^3 + x^2$ b e^{3x+1} by first differentiating e^{3x+1}
c \sqrt{x} by first differentiating $x\sqrt{x}$ d $(2x + 1)^3$ by first differentiating $(2x + 1)^4$.