18G – Integration by Substitution

- method to find an integral, but only when it can be set up in a certain way:

$$\int f(g(x))g'(x)dx = \int f(u)\frac{du}{dx}dx$$

$$= \int f(u)du$$

Examples:

(a) $\int \cos(x^2)2x\,dx$ Let $u = x^2$

$\frac{du}{dx} = 2x$ or $dx = \frac{du}{2x}$

$= \int \cos(u) \frac{du}{\cancel{dx}} \cdot \cancel{dx}$

$= \int \cos u \, du$

$= \sin u + C$

$= \sin(x^2) + C$

(b) $\int (x^3+2x)^5(3x^2+2)\,dx$

Let $u = x^3+2x$

$$\frac{du}{dx} = 3x^2+2$$

$$du = (3x^2+2)\,dx$$

$$= \int u^5\,du$$

$$= \frac{1}{6}u^6 + C$$

$$= \frac{1}{6}(x^3+2x)^6 + C$$

$$\int x^2\,dz$$

integrate with respect to z

constant

$$= x^2\int 1\,dz = x^2 z$$

(c) $\int \frac{x}{x^2+1}\,dx$

Let $u = x^2 + 1$

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\frac{du}{2x} = dx$$

$$= \int \frac{\cancel{x}}{u}\left(\frac{du}{2\cancel{x}}\right)$$

$$= \int \frac{1}{2u}\,du$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C \quad = \frac{1}{2}\ln|x^2+1| + C$$

(d) $\int xe^{x^2+2}\,dx$

let $u = x^2+2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{du}{2} = x\,dx$

$= \int e^u \cdot \frac{du}{2}$

$= \frac{1}{2}\int e^u\,du$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2+2} + C$

(e) Integrate with respect to x: $\sin^2 x \cos x$

$$\int \sin^2 x \cdot \cos x \cdot dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x\,dx$$

$$= \int (\sin x)^2 \cdot \cos x\,dx$$

$$= \int u^2\,du$$

$$= \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}\sin^3 x + C$$

(f) Integrate with respect to x: $\dfrac{\sin x}{\cos x}$

$$\int \frac{\sin x}{\cos x}\,dx$$

$$= \int \left(\frac{1}{\cos x}\right)\sin x\,dx$$

$$= \int \frac{1}{u}(-du)$$

$$= \int -\frac{1}{u}\,du = -\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

let $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x\,dx$$

$$-du = \sin x\,dx$$

let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x\,dx$$

$$\frac{du}{\cos x} = dx$$

let $u = \sin x$

$$\frac{du}{dx} = \cos x$$

$$du = \cos x\,dx$$

$$\frac{du}{\cos x} = dx$$

$$\int \frac{\sin x}{\cos x}\,dx$$

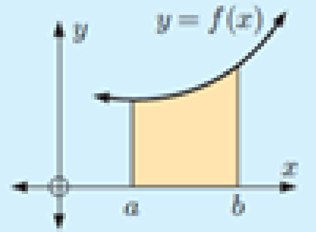
$$= \int \frac{u}{\cos x}\left(\frac{du}{\cos x}\right)$$

$$= \int \frac{u\,du}{\cos^2 x}$$

← no good because we want to get rid of x variable.

18H – Definite Integrals

If $f(x)$ is a continuous positive function on an interval $a \leqslant x \leqslant b$ then the area under the curve between $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$.



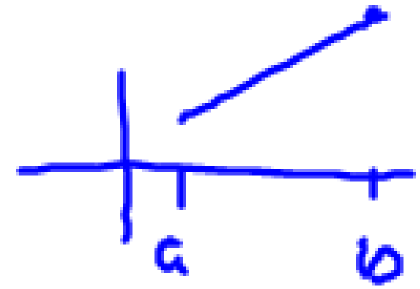
Fundamental Theorem of Calculus:

If $F(x)$ is the antiderivative of $f(x)$ where $f(x)$ is continuous on the interval $a \le x \le b$, then the **definite integral** of $f(x)$ on this interval is

$$\int_a^b f(x)dx = F(b) - F(a).$$

$\int_a^b f(x)dx$ reads "the integral from $x = a$ to $x = b$ of $f(x)$ with respect to x.

It is a **definite** integral because there are lower and upper limits for the integration, resulting in a numerical answer.



Notation:

Write $F(b)-F(a)$ as $[F(x)]_a^b$

So $$\int_a^b f(x)dx = [F(x)]_a^b = F(b)-F(a)$$

Properties: (pg. 468)

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
- $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, $\quad c$ is any constant
- $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
- $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

Examples:

Evaluate:

(a) $\int_1^5 (x^3 + 5x + 3)dx$

$= \frac{1}{4}x^4 + 5\left(\frac{1}{2}x^2\right) + 3x \Big]_1^5$

$= \left[\frac{1}{4}(5)^4 + \frac{5}{2}(5)^2 + 3(5)\right] - \left[\frac{1}{4}(1)^4 + \frac{5}{2}(1)^2 + 3(1)\right]$

$=$

(b) $\int_0^\pi \sin x dx$

$= -\cos x \Big]_0^\pi$

$= (-\cos\pi) - (-\cos(0))$

$= -(-1) - (-(1))$

$= 1 + 1$

$= 2$

$\int_0^\pi (\sin(X))dX$

2

(c) $\int_1^3 \frac{4}{x} + 3x^2 dx$

$= 4 \ln|x| + 3\left(\frac{1}{3}x^3\right) \Big]_1^3$

$= (4\ln 3 + (3)^3) - (4\ln(1) + (1)^3)$

$= 4\ln 3 + 27 - 1$

$= 4\ln 3 + 26$

Evaluating Integrals with Technology:

TI-84 Plus:

Press MATH

Choose 9: fnInt(

$$\int_{-1}^{1}(e^{x^2})dx$$

2.925303492

Evaluate $\int_{-1}^{1} e^{x^2}\,dx$

Hw on 18G +
18H