6 The function $y = e^x - 3e^{-x}$ cuts the x-axis at P and the y-axis at Q.

x-int y-int

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a Determine the coordinates of P and Q.

b Prove that the function is increasing for all x.

c Show that $\frac{d^2y}{dx^2} = y$. What can be deduced about the concavity of the function above and below the x-axis?

d Use technology to help graph $y = e^x - 3e^{-x}$.
Show the features of **a**, **b**, and **c** on the graph.

A) x-int (y=0)

$0 = e^x - 3e^{-x}$

$0 = e^x - \frac{3}{e^x}$

$0 = \frac{e^{2x} - 3}{e^x}$

$0 = e^{2x} - 3$

$e^{2x} = 3$

$\ln 3 = 2x$

$x = \frac{1}{2}\ln 3$

$(\frac{1}{2}\ln 3, 0)$

y-int (x=0)

$y = e^0 - 3e^{-0}$

$y = 1 - 3$

$y = -2$

$(0, -2)$

a Determine the coordinates of P and Q.

b Prove that the function is increasing for all x.

c Show that $\frac{d^2y}{dx^2} = y$. What can be deduced about the concavity of the function above and below the x-axis?

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b) to find slopes of tangents

$y = e^x - 3e^{-x}$ (Chain Rule)

$y' = e^x - 3(-e^{-x})$

$y' = e^x + 3e^{-x}$

$y' = 0$

$0 = e^x + \frac{3}{e^x}$

$0 = \frac{e^{2x} + 3}{e^x}$

VA: $e^x = 0$

never

$\therefore$ no VA

$0 = e^{2x} + 3$

$-3 = e^{2x}$ $2x = \ln(-3)$

not possible

Sign diagram for y'

$+$

test $x = 0$

c Show that $\frac{d^2y}{dx^2} = y$. What can be deduced about the concavity of the function above and below the x-axis?

d Use technology to help graph $y = e^x - 3e^{-x}$.
Show the features of **a**, **b**, and **c** on the graph.

$y = e^x - 3e^{-x}$

$y' = e^x + 3e^{-x}$

$y'' = e^x + 3(-e^{-x})$

$y'' = e^x - 3e^{-x}$

$y'' = y$

when $x > \frac{1}{2}\ln 3$, $y > 0$

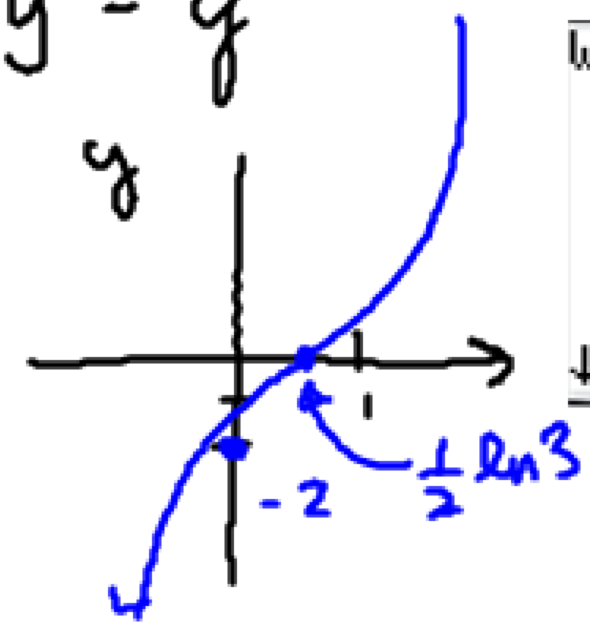
$y'' > 0$

↳ concave up

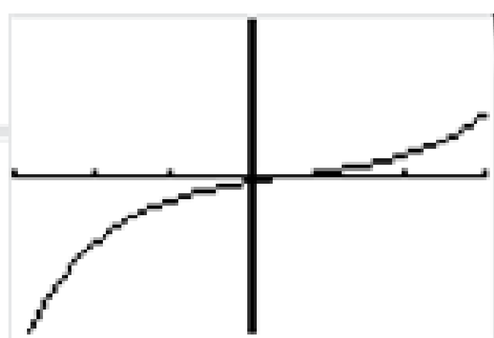
when $x < \frac{1}{2}\ln 3$ $y < 0$

$y'' < 0$

↳ concave down



WINDOW
Xmin=-3
Xmax=3
Xscl=1
Ymin=-50
Ymax=50
Yscl=1
↓Xres=1



9 Consider the function $f(x) = \dfrac{e^x}{x}$.

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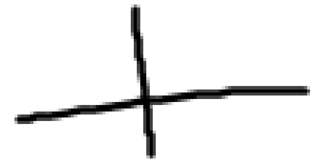


- **a** Does the graph of $y = f(x)$ have any x or y-intercepts?
- **b** Discuss $f(x)$ as $x \to \infty$ and as $x \to -\infty$.
- **c** Find and classify any stationary points of $y = f(x)$.
- **d** Find the intervals where $f(x)$ is: **i** concave up **ii** concave down.
- **e** Sketch the graph of $y = f(x)$ showing all important features.
- **f** Find the equation of the tangent to $f(x) = \dfrac{e^x}{x}$ at the point where $x = -1$.

A) x-int (y=0)

$0 = \dfrac{e^x}{x}$

$0 = e^x$

never

no x-int

y-int (x=0)

$f(0) = \dfrac{e^0}{0}$

Oh no we don't!!

no y-int

there must be a VA.

B) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{e^x}{x} = \infty$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{e^x}{x} = 0$

$\lim\limits_{x\to0^+} \dfrac{e^x}{x} = \infty$

$\lim\limits_{x\to0^-} \dfrac{e^x}{x} = -\infty$

c Find and classify any stationary points of $y = f(x)$.

d Find the intervals where $f(x)$ is: **i** concave up **ii** concave down.

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

c) $f'(x) = 0$ $\quad y = \frac{e^x}{x}$ ← Quotient Rule

$u = e^x \quad v = x$

$u' = e^x \quad v' = 1$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

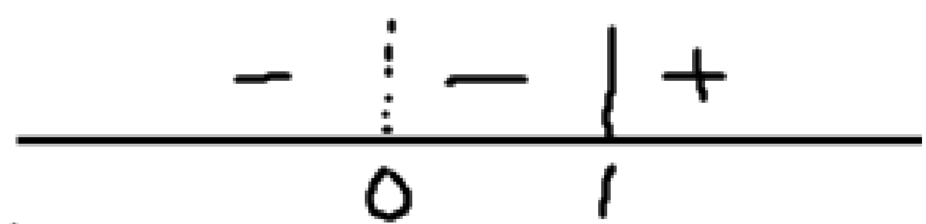
$$= \frac{(x)e^x - e^x(1)}{x^2}$$

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

$$0 = \frac{e^x(x-1)}{x^2}$$

→ VA: $x^2 = 0$, $x = 0$

x-int: $e^x = 0$ ← never $\qquad x - 1 = 0$, $x = 1$



test $x = 2$

$x = 1$ is a local minimum

→ $y = +e$

d Find the intervals where $f(x)$ is: **i** concave up **ii** concave down.

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

$f''(x) = 0$

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

Product Rule

$u = e^x(x-1)$ $\quad v = x^2$

$v' = 2x$

$u' = e^x(1) + (x-1)(e^x)$

$= e^x + xe^x - e^x$

$= xe^x$

$$f''(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2)(xe^x) - e^x(x-1)(2x)}{(x^2)^2}$$

$$= \frac{x^3e^x - 2x^2e^x + 2xe^x}{x^4}$$

$$f''(x) = \frac{xe^x(x^2-2x+2)}{x^4}$$

$$f''(x) = \frac{e^x(x^2-2x+2)}{x^3}$$

$$f''(x) = \frac{e^x(x^2-2x+2)}{x^3}$$

$$0 = \frac{e^x(x^2-2x+2)}{x^3}$$

VA: $x^3=0$ or $x=0$

x-int: $e^x=0$ ← never

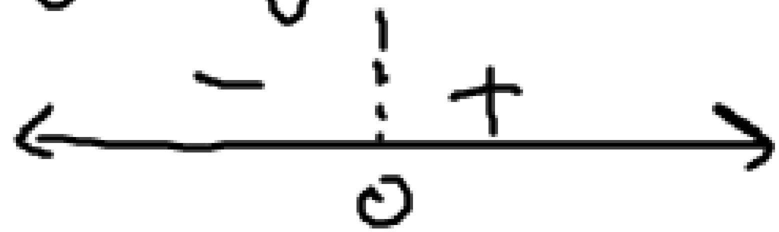
$x^2-2x+2=0$ ← never

$\Delta = b^2-4ac$ Discriminant

$\Delta<0$
no real roots

test
$x=1$

Sign diagram



CU: $x \in (0,\infty)$

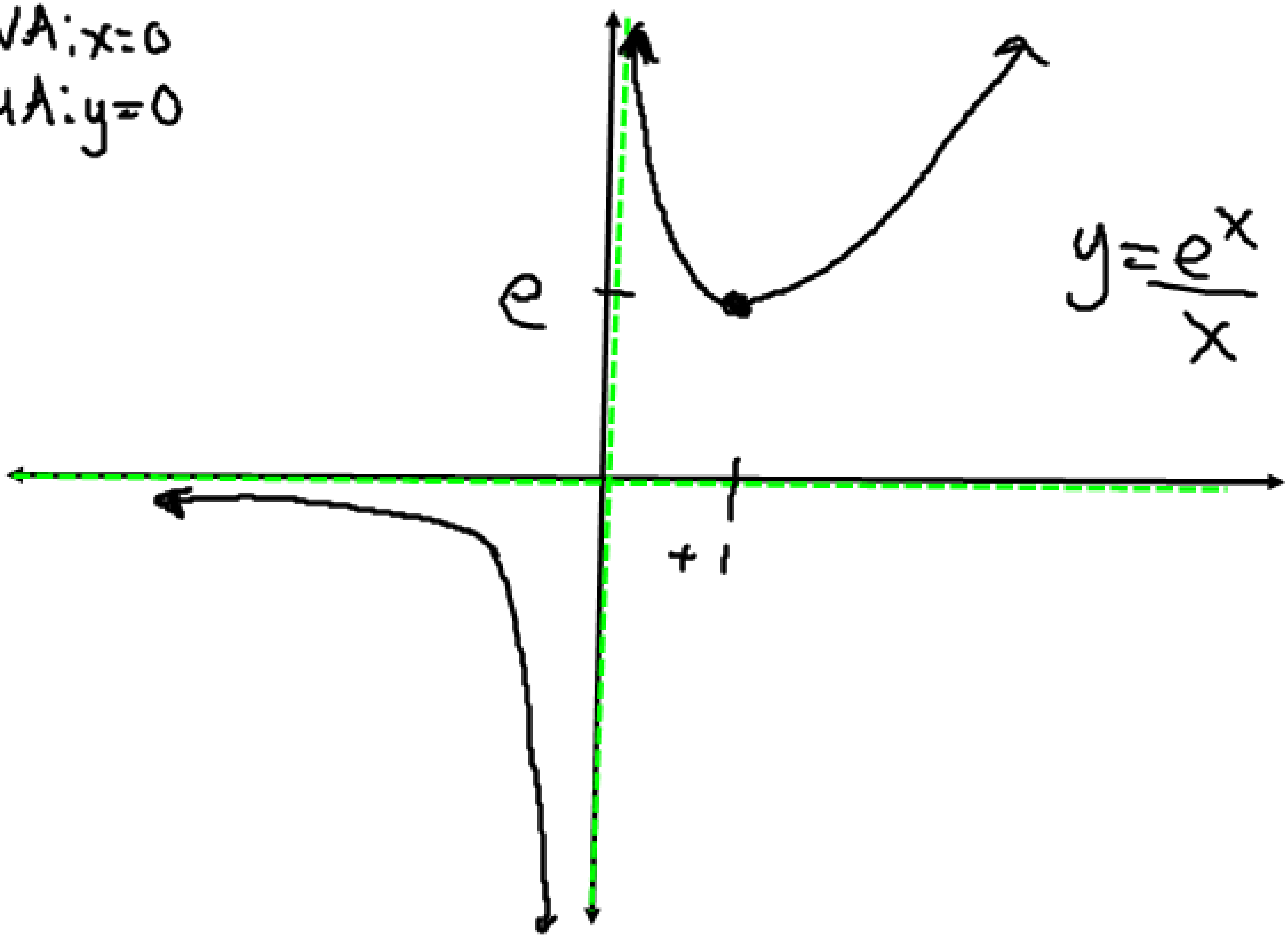
CD: $x \in (-\infty,0)$

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

VA: $x=0$

HA: $y=0$



f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

$$f(-1) = \frac{e^{-1}}{-1} = \frac{-1}{e}$$

$$f'(-1) = \frac{e^{-1}(-1-1)}{(-1)^2}$$

$$= \frac{-2}{e}$$

Slope of the tangent m_T

$$y = mx + b$$

$$\frac{-1}{e} = \left(\frac{-2}{e}\right)(-1) + b$$

$$\frac{-1}{e} = \frac{2}{e} + b$$

$$\frac{-3}{e} = b$$

$$y = \frac{-2}{e}x - \frac{3}{e}$$