ICA on ch 16 THIS THURSDAY AND FRIDAY

Ch16D Day 2

x-int

y-int

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6 The function $y = e^x - 3e^{-x}$ cuts the x-axis at P and the y-axis at Q.

a Determine the coordinates of P and Q.

b Prove that the function is increasing for all x.

c Show that $\frac{d^2y}{dx^2} = y$. What can be deduced about the concavity of the function above and below the x-axis?

d Use technology to help graph $y = e^x - 3e^{-x}$.
Show the features of **a**, **b**, and **c** on the graph.

A) x-int (y=0)

$0 = e^x - 3e^{-x}$

$0 = e^x - \frac{3}{e^x}$

$0 = \frac{e^{2x} - 3}{e^x}$

$0 = e^{2x} - 3$

$3 = e^{2x}$

$\ln 3 = 2x$

$x = \frac{1}{2}\ln 3$

$P(\frac{1}{2}\ln 3, 0)$

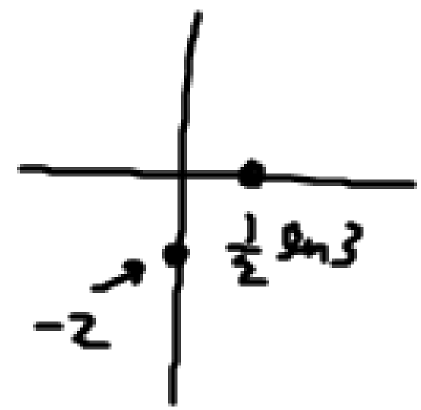
y-int (x=0)

$y = \frac{e^{2(0)} - 3}{e^0}$

$y = \frac{1-3}{1}$

$y = -2$

$Q(0, -2)$



B) find derivative

chain Rule

$y = e^x - 3e^{-x}$

$y' = e^x - 3(-e^{-x})$

$y' = e^x + 3e^{-x}$

$y' = \dfrac{e^{2x}+3}{e^x}$

$0 = \dfrac{e^{2x}+3}{e^x}$

$0 = e^{2x}+3$

$-3 = e^{2x}$

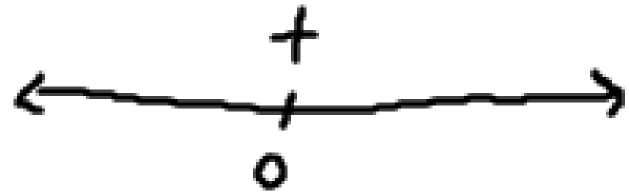
$2x = \ln(-3)$

not possible

$e^x = 0$ will never happen

→ no x-int, no Vertical Asymptotes

Sign diagram of $f'(x)$



Since the sign diagram of $f'(x)$ is always positive, the function is always increasing ✓

c Show that $\frac{d^2y}{dx^2} = y$. What can be deduced about the concavity of the function above and below the x-axis?

d Use technology to help graph $y = e^x - 3e^{-x}$.
Show the features of **a**, **b**, and **c** on the graph.

$y' = e^x + 3e^{-x}$

$y'' = e^x + 3(-e^{-x})$

$y'' = e^x - 3e^{-x}$

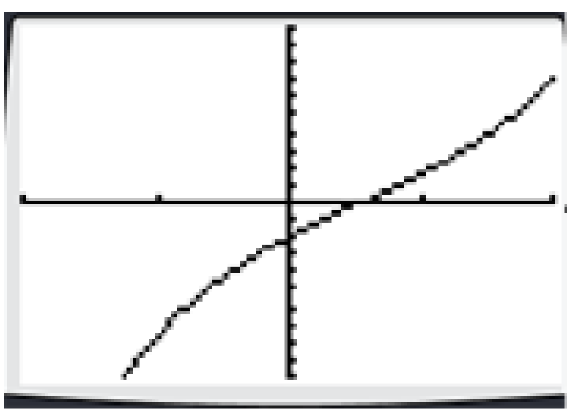
$y'' = y$

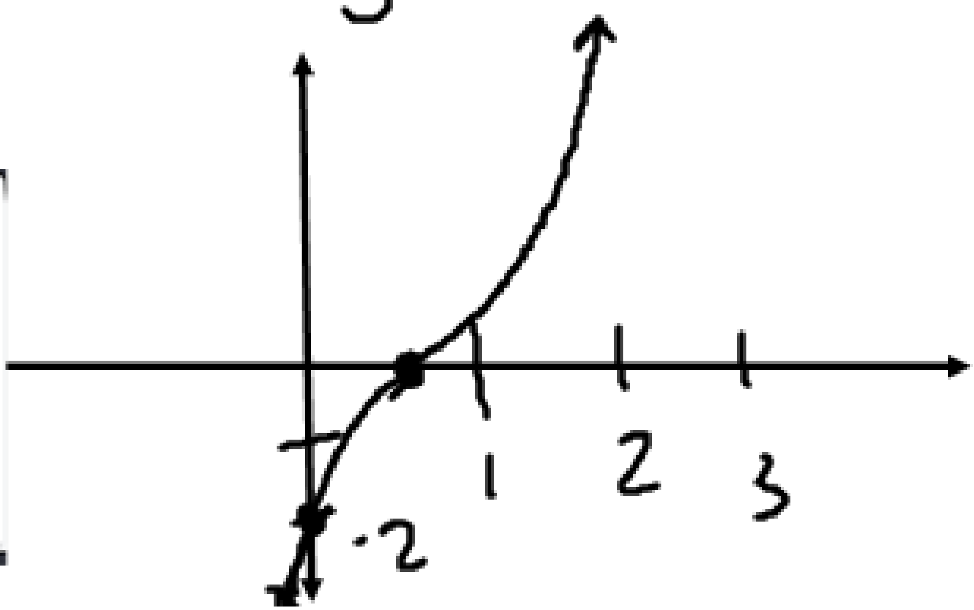
When $y > 0$, $x > \frac{1}{2}\ln 3$

$y'' > 0$ CU

When $y < 0$, $x < \frac{1}{2}\ln 3$

$y'' < 0$ CD





9 Consider the function $f(x) = \frac{e^x}{x}$.

a Does the graph of $y = f(x)$ have any x or y-intercepts?

b Discuss $f(x)$ as $x \to \infty$ and as $x \to -\infty$.

c Find and classify any stationary points of $y = f(x)$.

d Find the intervals where $f(x)$ is: i concave up ii concave down.

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

A)

x-int (y=0)

$0 = \frac{e^x}{x}$ or $0 = e^x$

NOT possible

no x int

y-int (x=0)

$f(0) = \frac{e^0}{0}$

Oh no we don't!

no y-int

There are no intercepts

B) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{e^x}{x} = \infty$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{e^x}{x} = 0$

talk about what's happening near x=0

$\lim_{x\to 0^+} \frac{e^x}{x} = \infty$ $\lim_{x\to 0^-} \frac{e^x}{x} = -\infty$

c Find and classify any stationary points of $y = f(x)$.

d Find the intervals where $f(x)$ is: **i** concave up **ii** concave down.

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

c) $f'(x) = 0$ $\quad f(x) = \frac{e^x}{x}$ $\quad u = e^x \quad v = x$

$u' = e^x \quad v' = 1$

Quotient Rule

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{xe^x - e^x}{x^2}$$

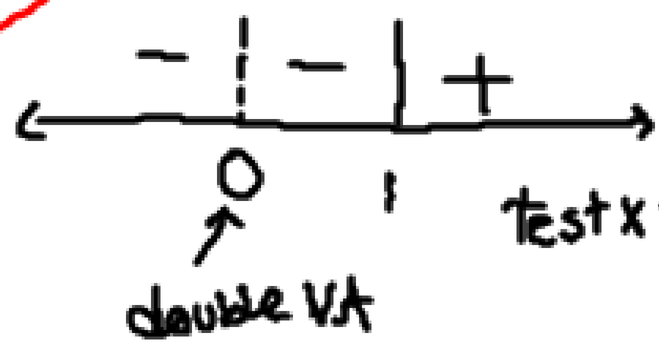
$$0 = \frac{e^x(x-1)}{x^2}$$

x-int

$e^x = 0$ never! $\quad x - 1 = 0$, $x = 1$

VA

$x^2 = 0$ or $x = 0$



test $x = 2$

$x = 1$ is local min

$$f(1) = \frac{e^1}{1} = e$$

local min at $(1, e)$

d Find the intervals where $f(x)$ is: **i** concave up **ii** concave down.

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

$f''(x)$

$$f'(x) = \frac{e^x(x-1)}{x^2}$$

← Product + Quotient Rule

$u = e^x(x-1)$ $\quad v = x^2$

$u' = e^x(1) + (x-1)e^x$ $\quad v' = 2x$

$= e^x + xe^x - e^x$

$= xe^x$

$$f''(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{(x^2)(xe^x) - (e^x(x-1))(2x)}{(x^2)^2}$$

$$= \frac{x^3e^x - 2x^2e^x + 2xe^x}{x^4}$$

$$f''(x) = \frac{xe^x(x^2-2x+2)}{x^4}$$

$$f''(x) = \frac{e^x(x^2-2x+2)}{x^3}$$

$$0 = \frac{e^x(x^2-2x+2)}{x^3}$$

$$0 = \frac{e^x(x^2-2x+2)}{x^3}$$

VA:

$x^3 = 0$

$x = 0$

x-int

$e^x = 0$ $\quad$ $x^2-2x+2=0$

never $\quad$ never

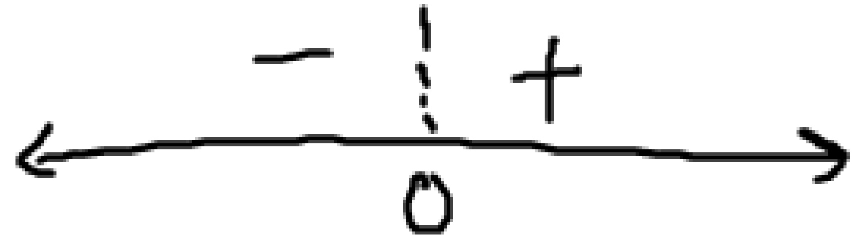
Quadratic formula

Discriminant

$\Delta = b^2-4ac$

$\Delta < 0$

Sign diagram for $f''(x)$



test $x=1$

CU: $x \in (0, \infty)$

CD: $x \in (-\infty, 0)$

e Sketch the graph of $y = f(x)$ showing all important features.

f Find the equation of the tangent to $f(x) = \dfrac{e^x}{x}$ at the point where $x = -1$.

no-intercepts

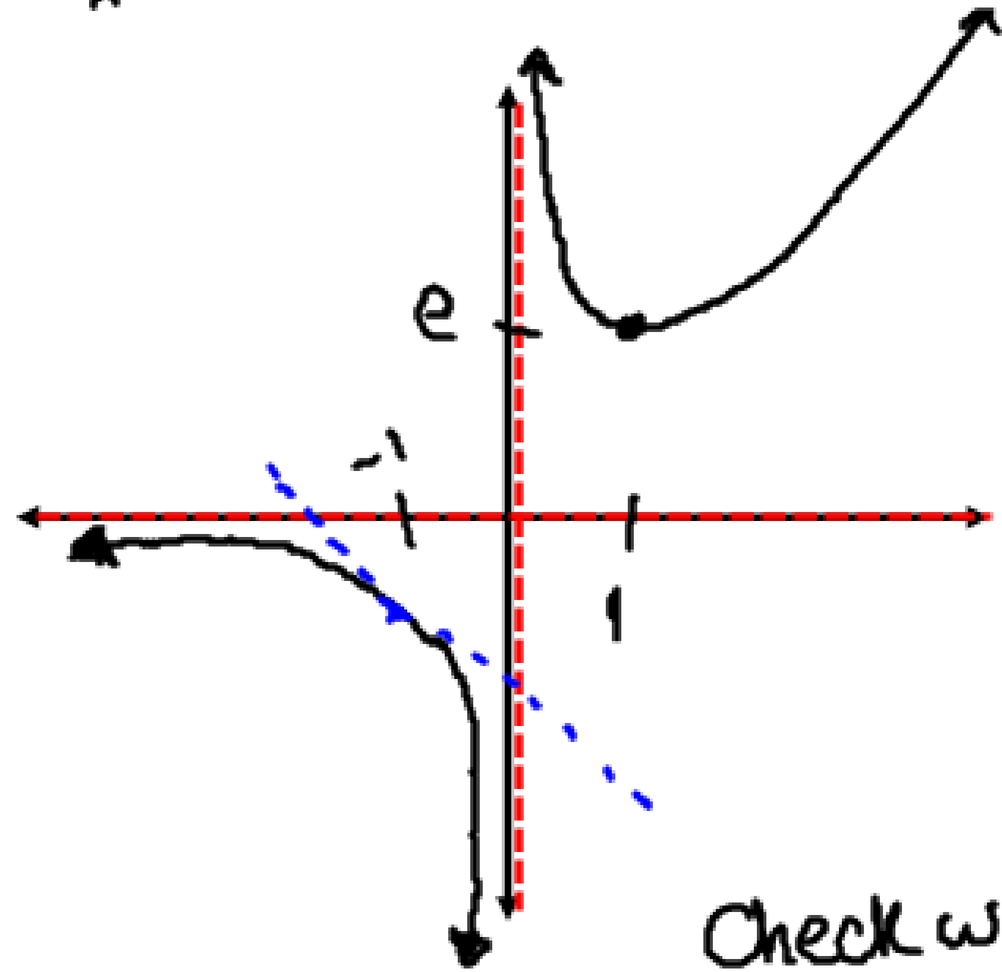
VA at $x=0$

HA at $y=0$

local min at $(1, e)$

CU $(0, \infty)$

CD $x \in (-\infty, 0)$



Check with TI84

f Find the equation of the tangent to $f(x) = \frac{e^x}{x}$ at the point where $x = -1$.

$f(-1) = \frac{e^{-1}}{-1} = \frac{-1}{e}$

$\left(-1, -\frac{1}{e}\right)$

$f'(x) = \frac{e^x(x-1)}{x^2}$

Slope of tangent

$f'(-1) = \frac{e^{-1}(-1-1)}{(-1)^2} = \frac{-2}{e}$

$y = mx + b$

$-\frac{1}{e} = \left(\frac{-2}{e}\right)(-1) + b$

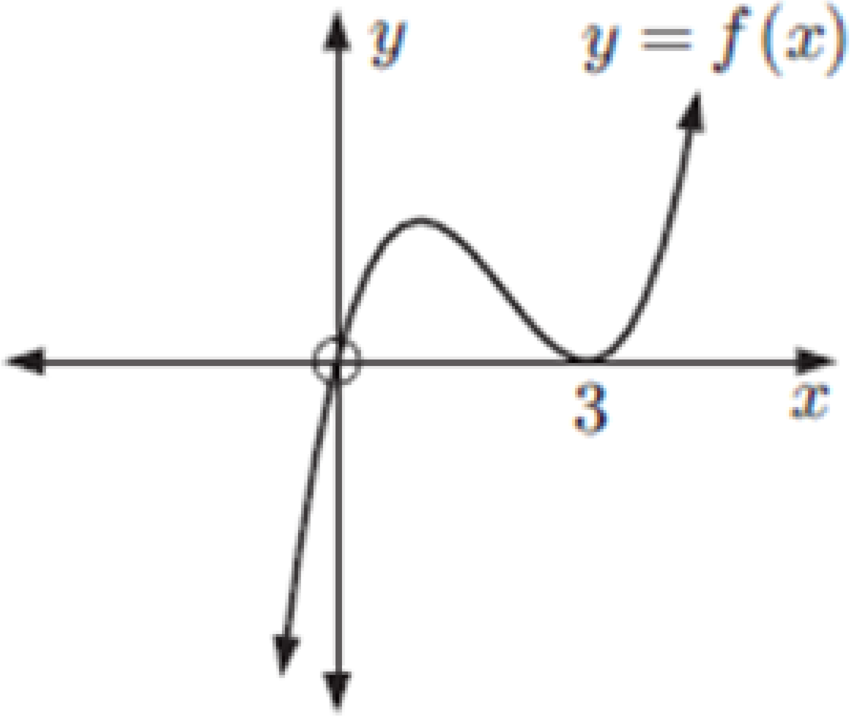
$-\frac{1}{e} = \frac{2}{e} + b$

$\frac{-3}{e} = b$

$y = -\frac{2}{e}x - \frac{3}{e}$

EXERCISE 16D.2

1 Using the graphs of $y = f(x)$ below, sketch the graphs of $y = f'(x)$ and $y = f''(x)$. Show
...ts.



f(x) is a cubic (degree 3)

$f(x) = ax^3 + \cdots$

f'(x) is a quadratic

f''(x) is linear

slopes of tangent lines

read these examples

textbook

2 For the graphs of $y = f'(x)$ below, sketch a graph which could be $y = f(x)$. Show clearly the location of any stationary points and points of inflection.

b