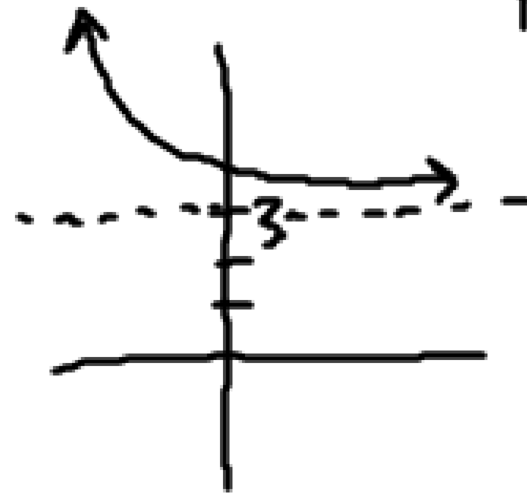
Ch16B pg396 2K

ch3 + ch5 VT+3
R_y

$f(x) = 3 + e^{-x}$

$f(x) = e^{-x} + 3$



$f'(x) = (e^{-x})(-1) + 0$

$f'(x) = -e^{-x}$

$0 = -e^{-x}$

never....

dec: $x \in R$

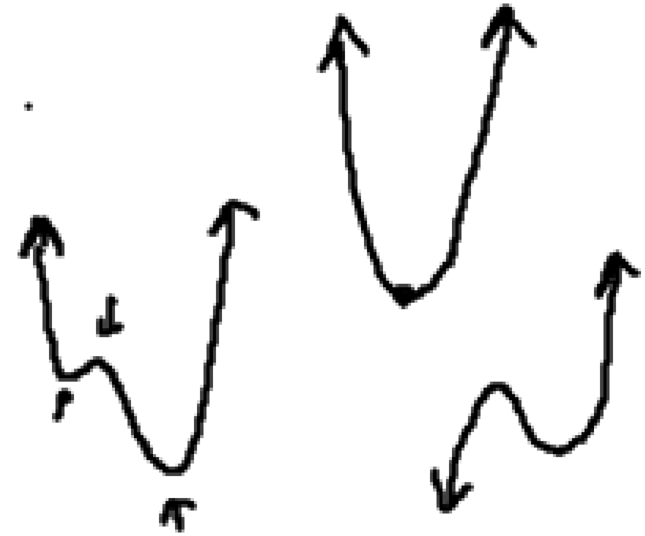
inc: never.

$f(1) = -e^{-1} = \frac{-1}{e} < 0$

$f(-1) = -e^{-(-1)} = -e < 0$

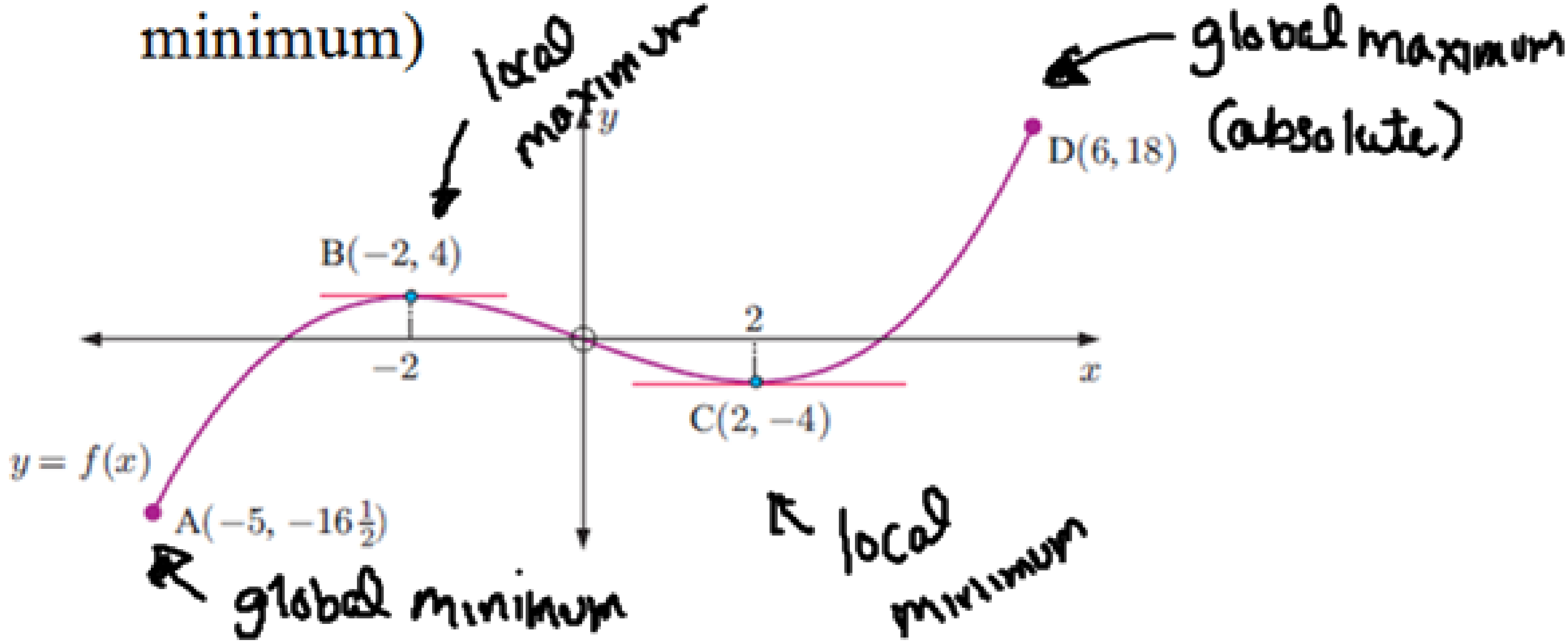
16C – Stationary Points

A stationary point is where $f'(x) = 0$



This can happen in two situations:

1) The graph has a turning point (local maximum or minimum)



2) The graph has an inflection

$$f(x)=(x-3)^3+1$$

← HT +3
VT −1
from $y=x^3$

$f'(x)=3(x-3)^2+0$

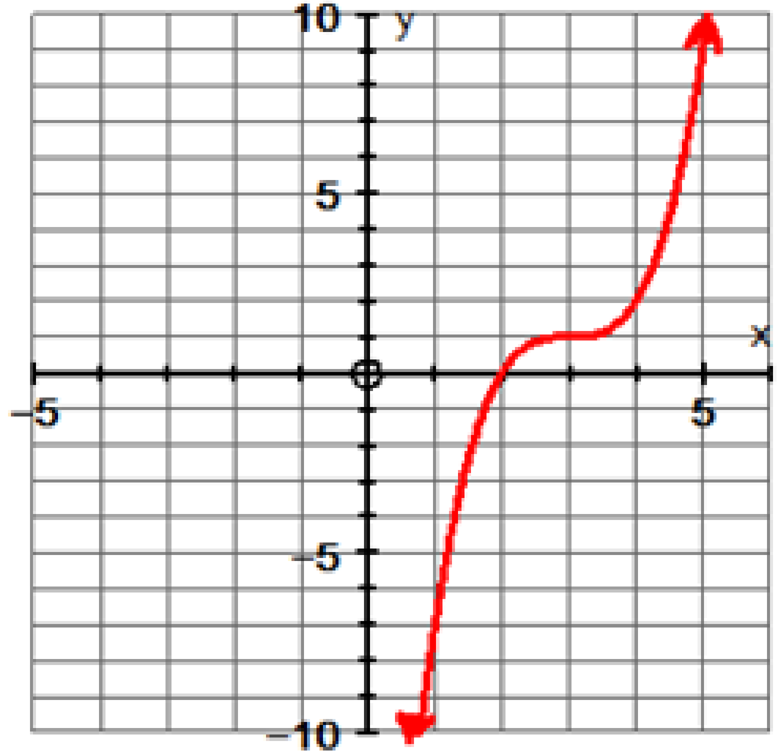
$0=3(x-3)^2$

$0=(x-3)^2$

$x=3$ ← repeats twice

+ | +

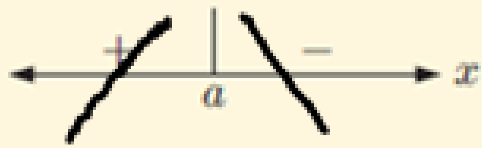
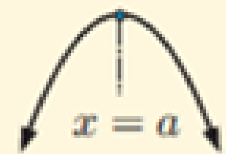
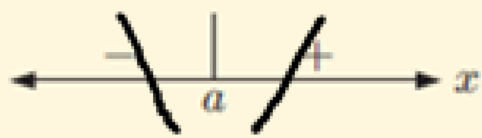
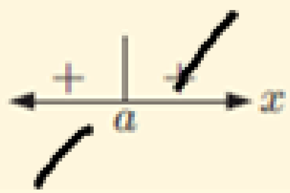
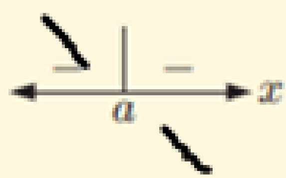
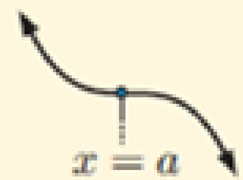
0 3



$f(x)=x^3+3(x)^2(-3)+3(x)(-3)^2+(-3)^3+1$

$f(x)=x^3-9x^2+27x-26$

Sign Diagrams and Stationary Points

Stationary point where $f'(a) = 0$	*Sign diagram of* $f'(x)$ *near* $x = a$	*Shape of curve near* $x = a$
local maximum		
local minimum		
stationary inflection	 or 	 or 

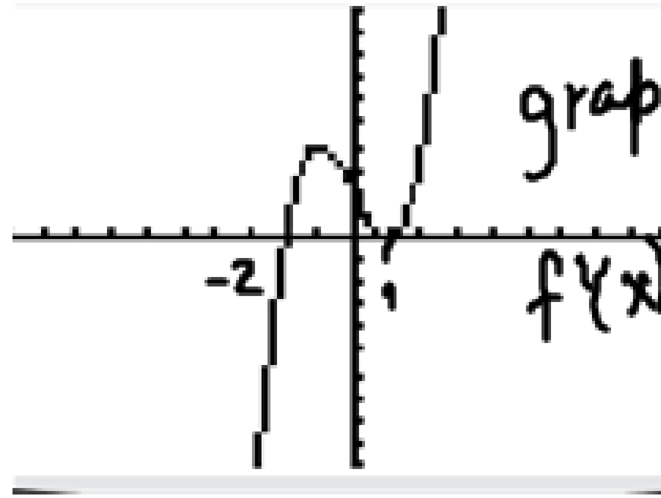
Example: Find and classify all stationary points. Sketch the function, labelling all important features (#2h pg 400)

$f(x) = x^4 - 6x^2 + 8x - 3$

$f'(x) = 4x^3 - 12x + 8$

$0 = 4x^3 - 12x + 8$

$0 = x^3 - 3x + 2$

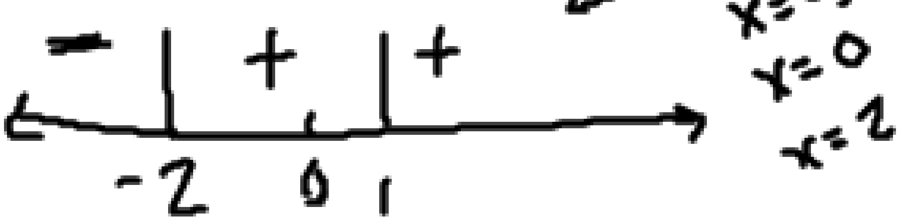


hmm...... graphing tech.

$x = -2 \quad x = 1$

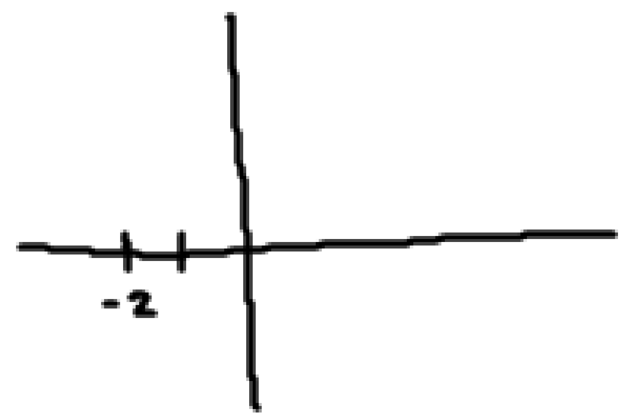
double root

$0 = (x+2)(x-1)(x-1)$



$x = -2$ ← local min

$x = 1$ ← inflection pt.



When $x=-2$

$f(-2) = (-2)^4 - 6(-2)^2 + 8(-2) - 3$

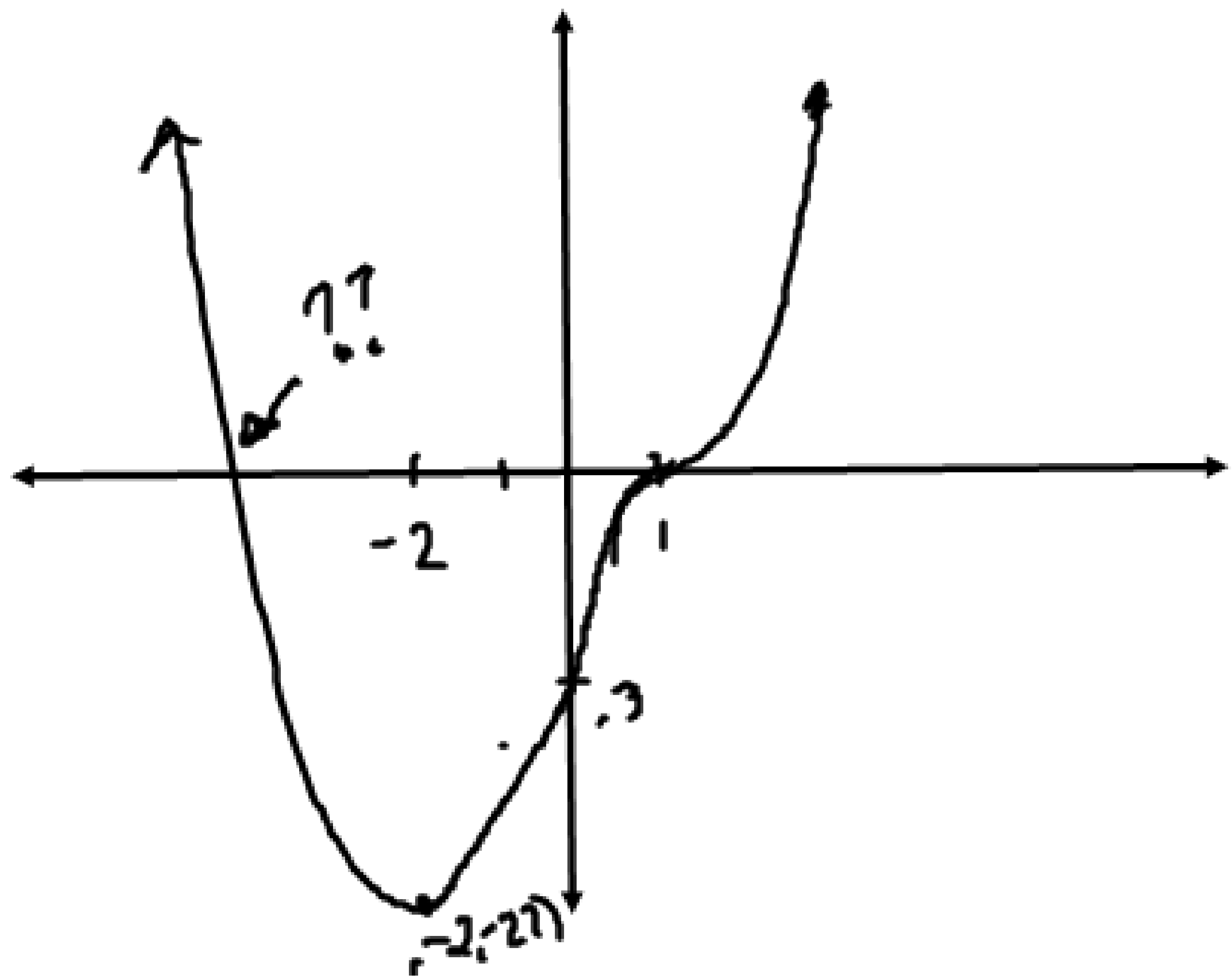
$= 16 - 24 - 16 - 3$

$f(-2) = -27$

When $x = 1$

$f(1) = 1 - 6 + 8 - 3$

$= 0$



Example: $f(x) = x^3 - 6x^2 + ax + b$ has a stationary point at $(2,1)$ ← $f'(x)=0$ here because it is a stationary pt.

A) Find the values of a and b

B) Find the position and nature of the stationary point

A)

$f(2)=1$

$1=(2)^3-6(2)^2+a(2)+b$

$1=8-24+2a+b$

$1=-16+2a+b$

$17=2a+b$

$f'(x)=3x^2-12x+a$

$f'(2)=3(2)^2-12(2)+a$

$0=12-24+a$

$12=a$

$\hookrightarrow b=17-2a$

$=17-2(12)$

$b=-7$

B) $f(x)=x^3-6x^2+12x-7$

$f'(x)=3x^2-12x+12$ ←

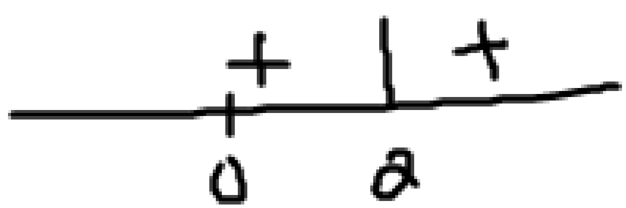
B) $f(x) = x^3 - 6x^2 + 12x - 7$

$f'(x) = 3x^2 - 12x + 12$

$0 = 3x^2 - 12x + 12$

$0 = x^2 - 4x + 4$

$0 = (x-2)^2 \quad x = 2$



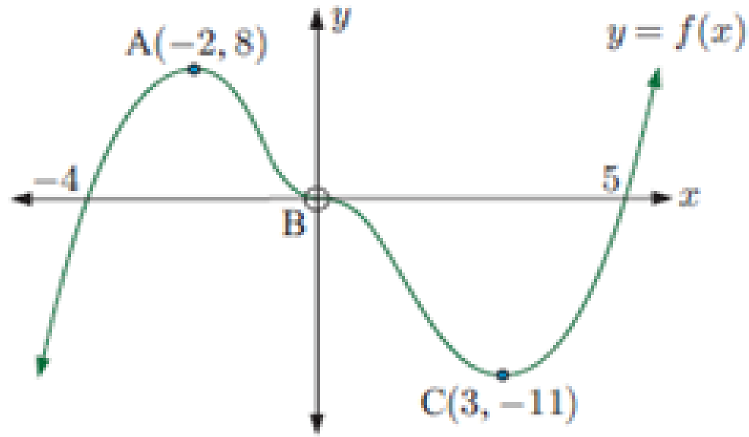
test $x = 0$ on $f'(x)$

$x = 3$

$(2,1)$ is an inflection pt.

EXERCISE 16C

1 The tangents at points A, B and C are horizontal.



a Classify points A, B and C.

b Draw a sign diagram for the gradient function $f'(x)$ for all x.

c State intervals where $y = f(x)$ is:

i increasing **ii** decreasing.

d Draw a sign diagram for $f(x)$ for all x.

2 For each of the following functions, find and classify any stationary points. Sketch the function, showing all important features.

a $f(x) = x^2 - 2$

b $f(x) = x^3 + 1$

c $f(x) = x^3 - 3x + 2$

d $f(x) = x^4 - 2x^2$

e $f(x) = x^3 - 6x^2 + 12x + 1$

f $f(x) = \sqrt{x} + 2$

g $f(x) = x - \sqrt{x}$

h $f(x) = x^4 - 6x^2 + 8x - 3$

i $f(x) = 1 - x\sqrt{x}$

j $f(x) = x^4 - 2x^2 - 8$

3 At what value of x does the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$, have a stationary point? Under what conditions is the stationary point a local maximum or a local minimum?

4 Find the position and nature of the stationary point(s) of:

a $y = xe^{-x}$

b $y = x^2e^x$

c $y = \dfrac{e^x}{x}$

d $y = e^{-x}(x + 2)$

5 $f(x) = 2x^3 + ax^2 - 24x + 1$ has a local maximum at $x = -4$. Find a.

6 $f(x) = x^3 + ax + b$ has a stationary point at $(-2, 3)$.

- **a** Find the values of a and b.
- **b** Find the position and nature of all stationary points.

7 Consider $f(x) = x \ln x$.

- **a** For what values of x is $f(x)$ defined?
- **b** Show that the global minimum value of $f(x)$ is $-\frac{1}{e}$.

8 For each of the following, determine the position and nature of the stationary points on the interval $0 \leqslant x \leqslant 2\pi$, then show them on a graph of the function.

- **a** $f(x) = \sin x$
- **b** $f(x) = \cos(2x)$
- **c** $f(x) = \sin^2 x$
- **d** $f(x) = e^{\sin x}$
- **e** $f(x) = \sin(2x) + 2\cos x$

9 The cubic polynomial $P(x) = ax^3 + bx^2 + cx + d$ touches the line with equation $y = 9x + 2$ at the point $(0, 2)$, and has a stationary point at $(-1, -7)$. Find $P(x)$.

10 Find the greatest and least value of:

- **a** $x^3 - 12x - 2$ for $-3 \leqslant x \leqslant 5$
- **b** $4 - 3x^2 + x^3$ for $-2 \leqslant x \leqslant 3$

11 Show that $y = 4e^{-x} \sin x$ has a local maximum when $x = \frac{\pi}{4}$.