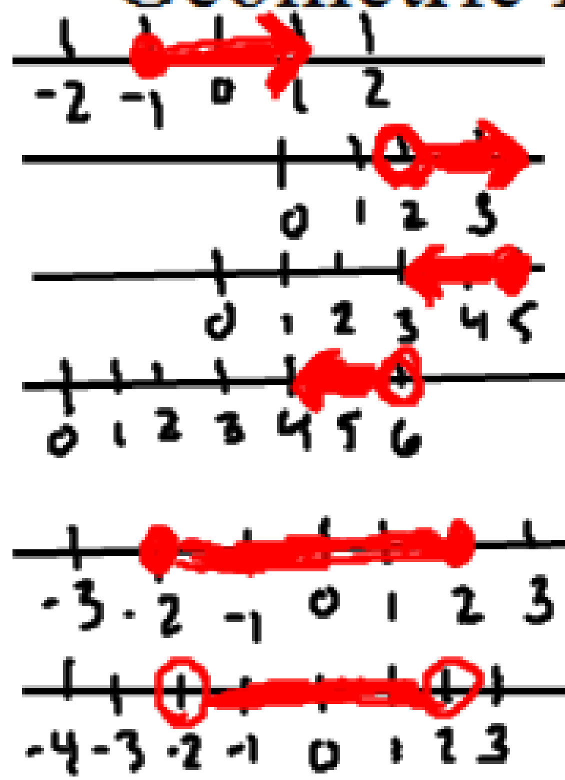
16B- Increasing and Decreasing Functions

Review of interval notation:

Algebraic Form ← include

Geometric Form



$x \geq -1$ $\quad x \in [-1, \infty)$ ← not included

$x > 2$ $\quad x \in (2, \infty)$

$x \leq 5$ $\quad x \in (-\infty, 5]$

$x < 6$ $\quad x \in (-\infty, 6)$

$-2 \leq x \leq 2$ $\quad x \in [-2, 2]$

$-2 < x < 2$ $\quad x \in (-2, 2)$

$x \in]-2, 2[$

IB ↖ not included

Increasing Functions: As the values of x increase, the values of y increase.

Decreasing Functions: As the values of x increase, the values of y decrease.

Example: Identify the intervals of increase and interval of decrease for the following graph:

union

increasing $x \in (-\infty, -1) \cup (1, \infty)$

$x < -1$ and $x > 1$

decreasing $x \in (-1, 1)$

What do you notice about the slope of the tangents on these intervals?

increasing: slope of tangent is positive $f'(x) > 0$

decreasing: slope of tangent is negative $f'(x) < 0$

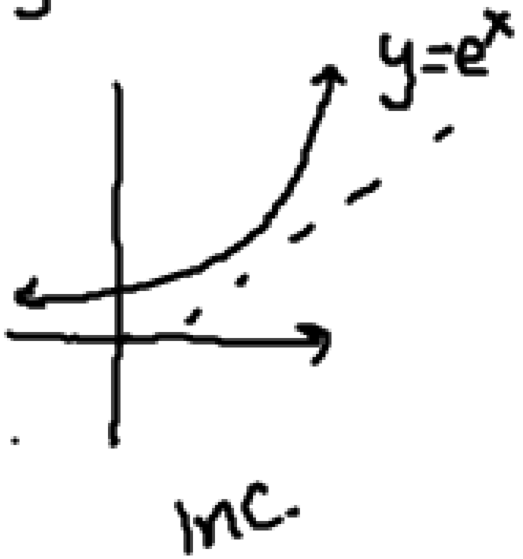
at $f'(x) = 0$ ← horizontal tangent

slope $= 0$ neither increasing nor decreasing

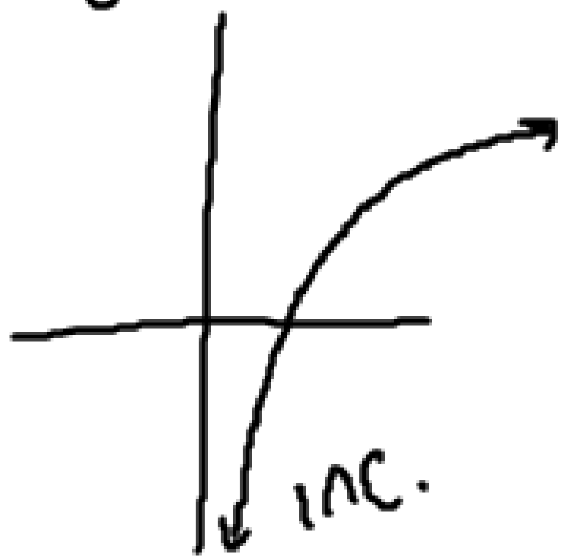
There are some graphs that will always be increasing or decreasing. These are called monotone increasing/decreasing functions:

Example: Exponential/logarithmic Functions

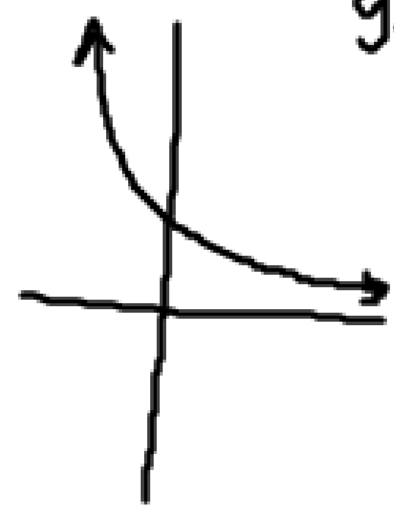
$y = e^x$



$y = \ln x$ (inverse of $y=e^x$)



$y = \left(\frac{1}{2}\right)^x$ or $y = 2^{-x}$



Example: Find the interval of increase and decrease for the following:

A) $f(x) = 2x^3 + 3x^2 - 12x - 9$

$f'(x) = 6x^2 + 6x - 12$

find where $f'(x) = 0$

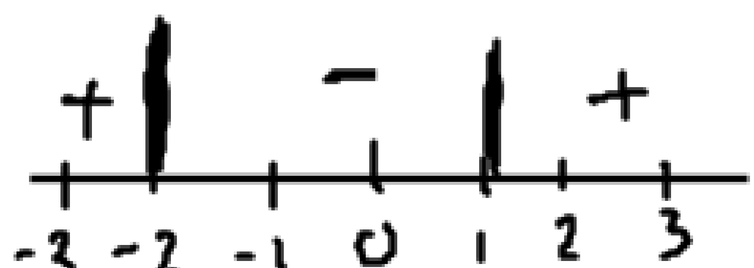
$0 = 6x^2 + 6x - 12$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2 \quad x = 1$

<u>Sign diagrams</u> (ch 2)



test $x = 0$

$x = 2$

$x = -3$

Inc: $x \in (-\infty, -2) \cup (1, \infty)$

dec: $x \in (-2, 1)$

B) $f(x)=3x-6\sqrt{x}+1$ $\qquad \sqrt{x} = x^{\frac{1}{2}}$

$$f'(x) = 3-6\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= 3-3x^{-\frac{1}{2}}$$

$$= 3-\frac{3}{\sqrt{x}}$$

$$f'(x) = \frac{3\sqrt{x}-3}{\sqrt{x}} \longrightarrow f'(x)=0$$

$$0 = \frac{3\sqrt{x}-3}{\sqrt{x}}$$

Asymptotes + x-intercepts

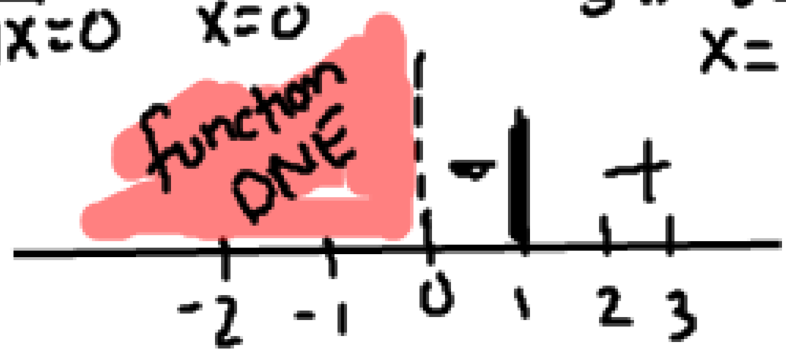
denominator

$\sqrt{x}=0 \quad x=0$

numerator

$3\sqrt{x}-3=0$

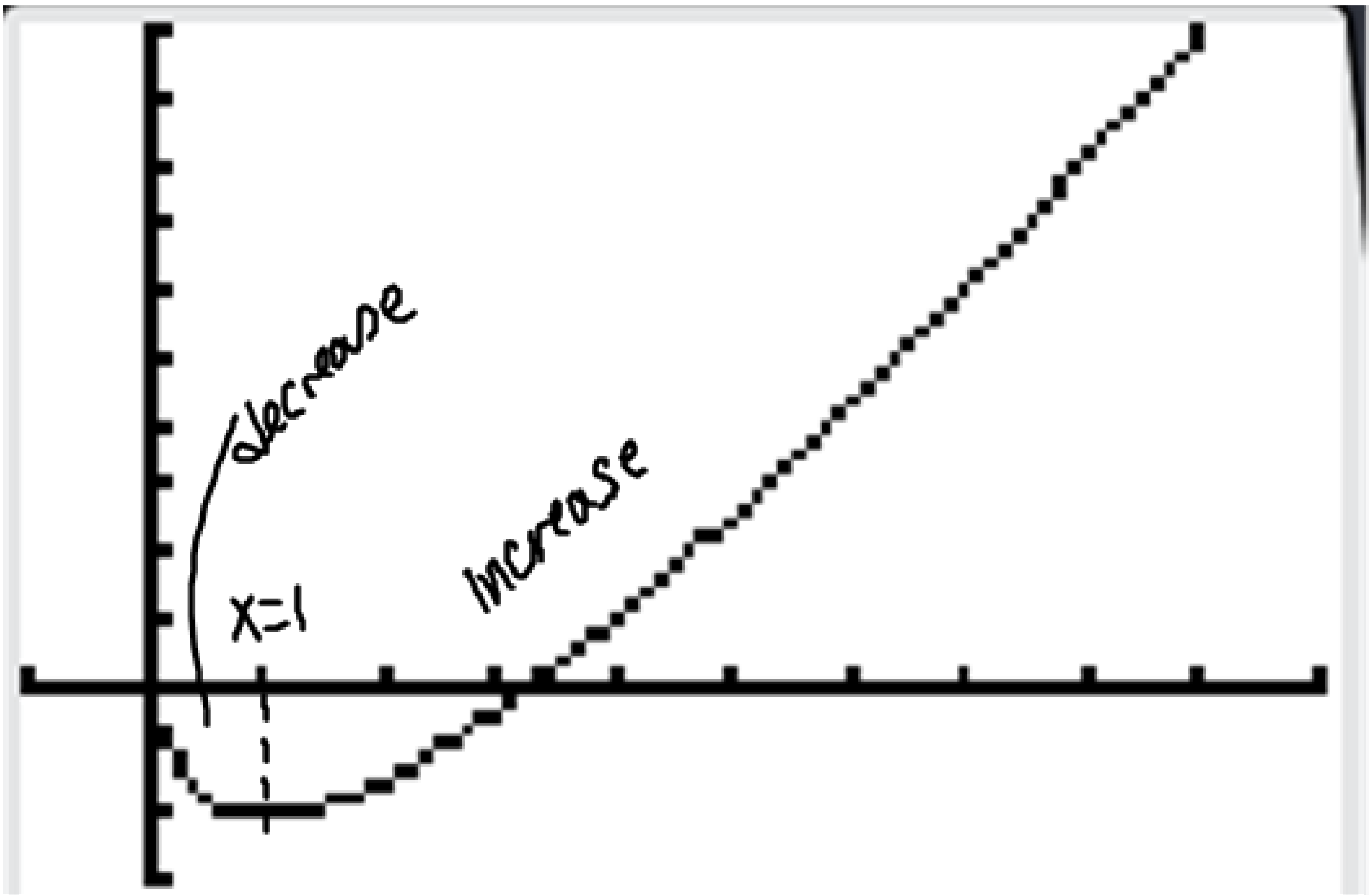
$x=1$



test pt $x=4$

Inc: $x\in(1,\infty)$

dec: $x\in(0,1)$



decrease
Increase
x=1

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3 Consider $f(x) = \dfrac{4x}{x^2+1}$.

a Show that $f'(x) = \dfrac{-4(x+1)(x-1)}{(x^2+1)^2}$ and draw its sign diagram.

b Hence, find intervals where $y = f(x)$ is increasing or decreasing.

$u = 4x \quad v = x^2+1$

$u' = 4 \quad v' = 2x$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$f'(x) = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2}$$

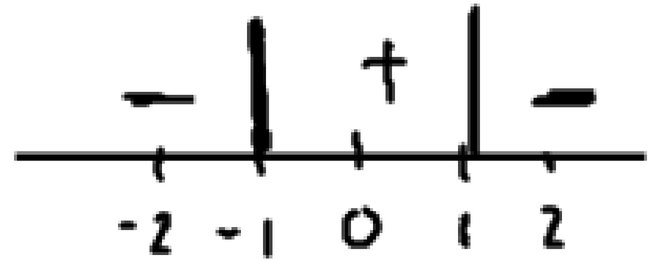
$$f'(x) = \frac{4x^2+4-8x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{-(4x^2-4)}{(x^2+1)^2}$$

$$= \frac{-4(x^2-1)}{(x^2+1)^2}$$

$$f'(x) = \frac{-4(x+1)(x-1)}{(x^2+1)^2}$$

$$f'(x) = \frac{-4(x+1)(x-1)}{(x^2+1)^2}$$



x-int: $-4(x+1)(x-1) = 0$

$x = 1$, $x = -1$

test pt: $x = 0$

VA: $(x^2+1)^2 = 0$ $\quad x^2+1=0$

$x^2 = -1$

no sol'n

$$f'(0) = \frac{-4(1)(-1)}{(1)^2} = +$$

No values of x that will make the denominator equal zero

Increasing: $x \in (-1, 1)$

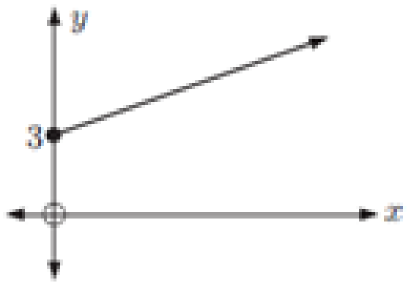
decreasing: $x \in (-\infty, -1) \cup (1, \infty)$

HW ch 16B

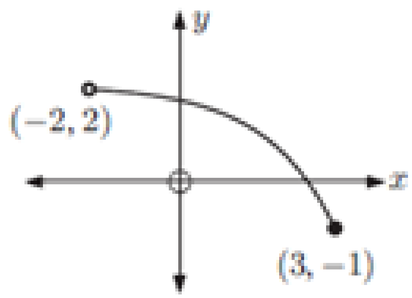
EXERCISE 16B

1 Write down the intervals where the graphs are: **i** increasing **ii** decreasing.

a



b



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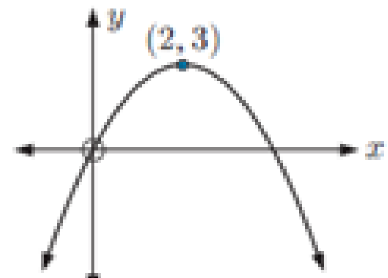

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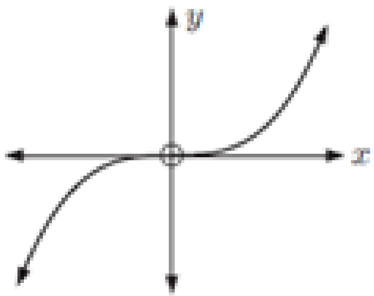

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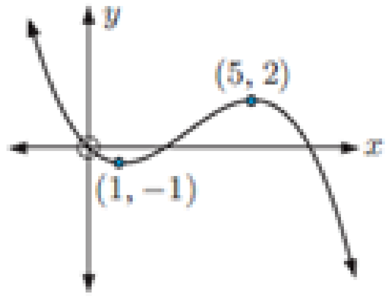

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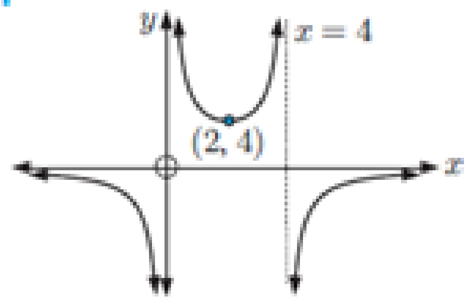

2 Find the intervals where $f(x)$ is increasing or decreasing:

a $f(x) = x^2$

b $f(x) = -x^3$

c $f(x) = 2x^2 + 3x - 4$

d $f(x) = \sqrt{x}$

e $f(x) = \dfrac{2}{\sqrt{x}}$

f $f(x) = x^3 - 6x^2$

g $f(x) = e^x$

h $f(x) = \ln x$

i $f(x) = -2x^3 + 4x$

j $f(x) = -4x^3 + 15x^2 + 18x + 3$

k $f(x) = 3 + e^{-x}$

l $f(x) = xe^x$

m $f(x) = 3x^4 - 16x^3 + 24x^2 - 2$

n $f(x) = 2x^3 + 9x^2 + 6x - 7$

o $f(x) = x^3 - 6x^2 + 3x - 1$

p $f(x) = x - 2\sqrt{x}$

4 Consider $f(x) = \dfrac{4x}{(x-1)^2}$.

a Show that $f'(x) = \dfrac{-4(x+1)}{(x-1)^3}$ and draw its sign diagram.

b Hence, find intervals where $y = f(x)$ is increasing or decreasing.

5 Consider $f(x) = \dfrac{-x^2+4x-7}{x-1}$.

a Show that $f'(x) = \dfrac{-(x+1)(x-3)}{(x-1)^2}$ and draw its sign diagram.

b Hence, find intervals where $y = f(x)$ is increasing or decreasing.

6 Find intervals where $f(x)$ is increasing or decreasing if:

a $f(x) = \dfrac{x^3}{x^2-1}$

b $f(x) = e^{-x^2}$

c $f(x) = x^2 + \dfrac{4}{x-1}$

d $f(x) = \dfrac{e^{-x}}{x}$