2 For each of the following functions, find and classify any stationary points. Sketch the function, showing all important features.

when $f'(x) = 0$

j $f(x) = x^4 - 2x^2 - 8$

$f'(x) = 4x^3 - 4x$

$0 = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$0 = 4x(x+1)(x-1)$

$x = 0 \quad x = -1 \quad x = 1$

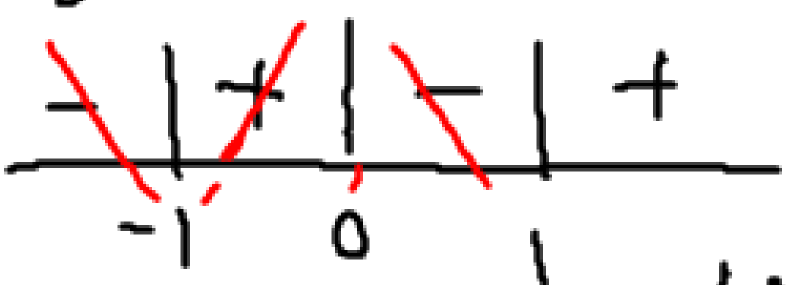
The slope of the tangent is 0 here (horizontal)

at $x = -1$ local min $(-1, -9)$

at $x = 0$ local max $(0, -8)$

at $x = 1$ local min $(1, -9)$

Sign diagram for $f'(x)$



test pt $x = 2$

If ....

$x = 3$ is an inflection

i $f(x) = x^4 - 2x^2 - 8$

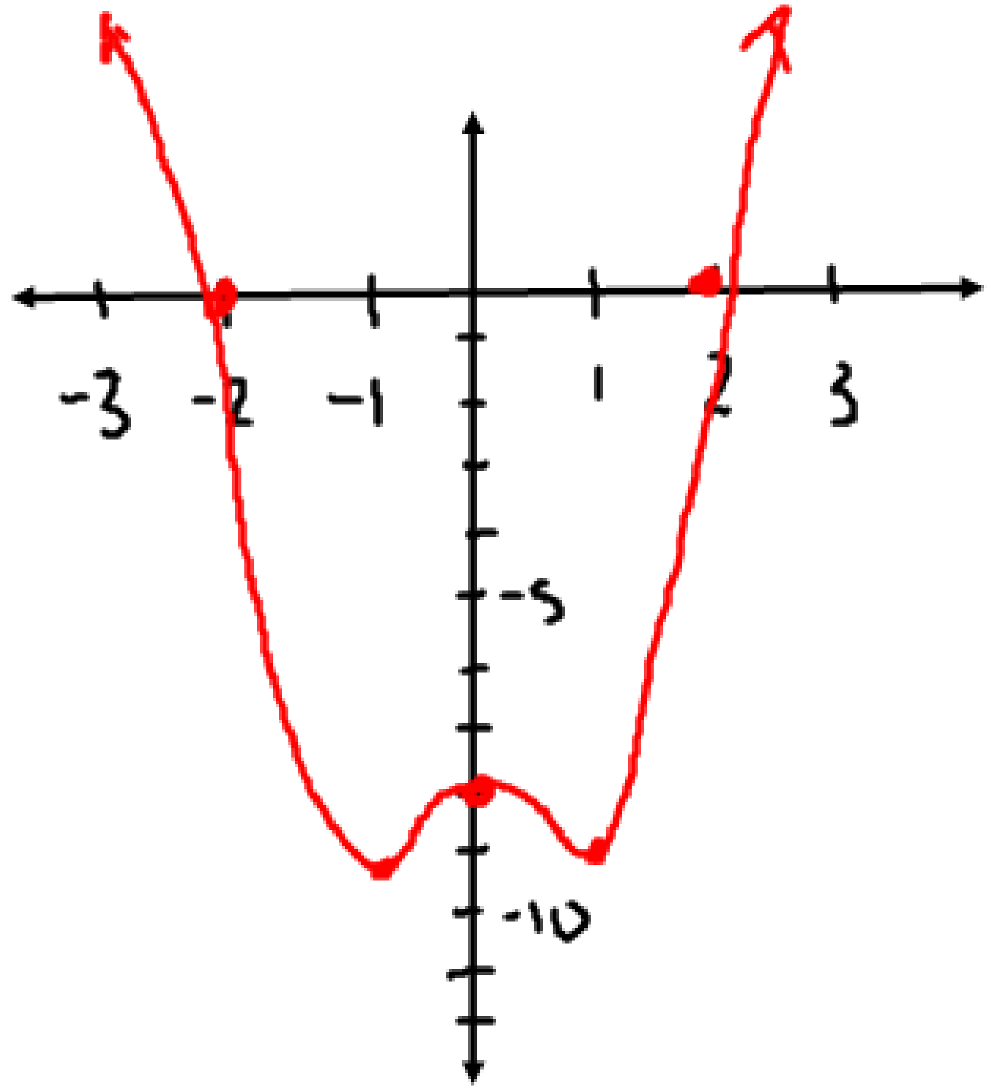
x-int:

$0 = x^4 - 2x^2 - 8$

$= (x^2 - 4)(x^2 + 2)$

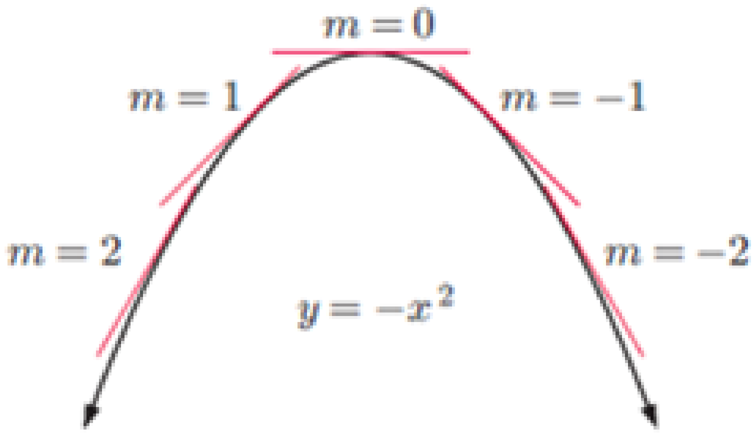
$= (x+2)(x-2)(x^2+2)$

$x = -2, 2$



Ch 16 D Inflections and Shape Day 1

Concave Down:



← the curve is below the tangent line

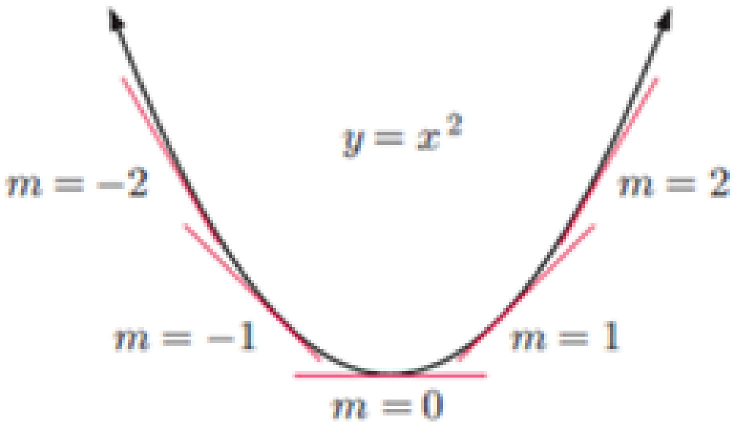
As x increases, the slope of the tangent decreases
This means the derivative of the tangent (double derivative) is negative

Example: $f(x) = -x^2$

$f'(x) = -2x$

$f''(x) = -2$

Concave up:



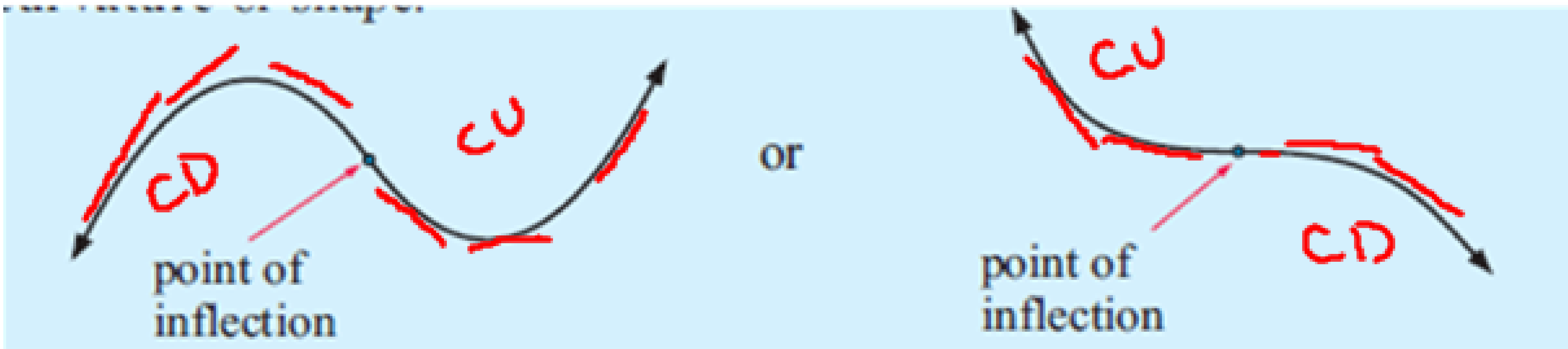
← The curve is above the tangent lines

As x increases, the slope of the tangent increases
This means the derivative of the tangent (double derivative) is positive

Example: $f(x) = x^2$ $\quad$ $f'(x) = 2x$

$f''(x) = 2$

Points of Inflection- a change in the curvature (going from concave up to concave down and vice versa)



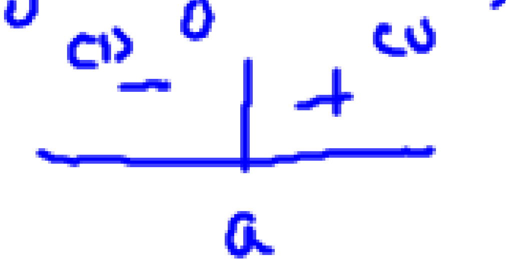
Two types of inflection points:

1- Stationary inflection (last section)

When the slope of the tangent is horizontal ($f'(x)=0$)

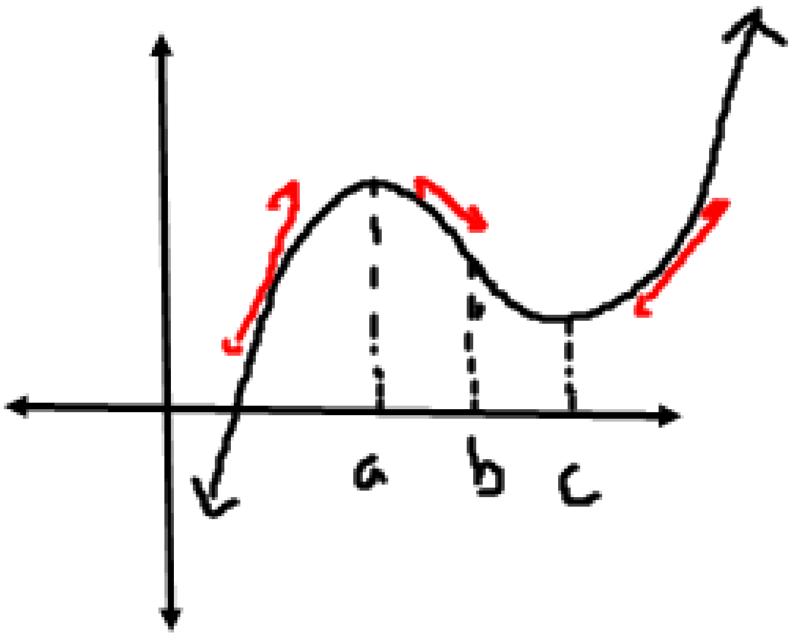
Sign diagram for $f'(x)$

Sign diagram for $f''(x)$



2- Non-stationary Inflection point

When the slope of the tangent is not horizontal ($f'(x) \neq 0$)



Sign diagram for $f'(x)$

$+$	a	$-$	c	$+$

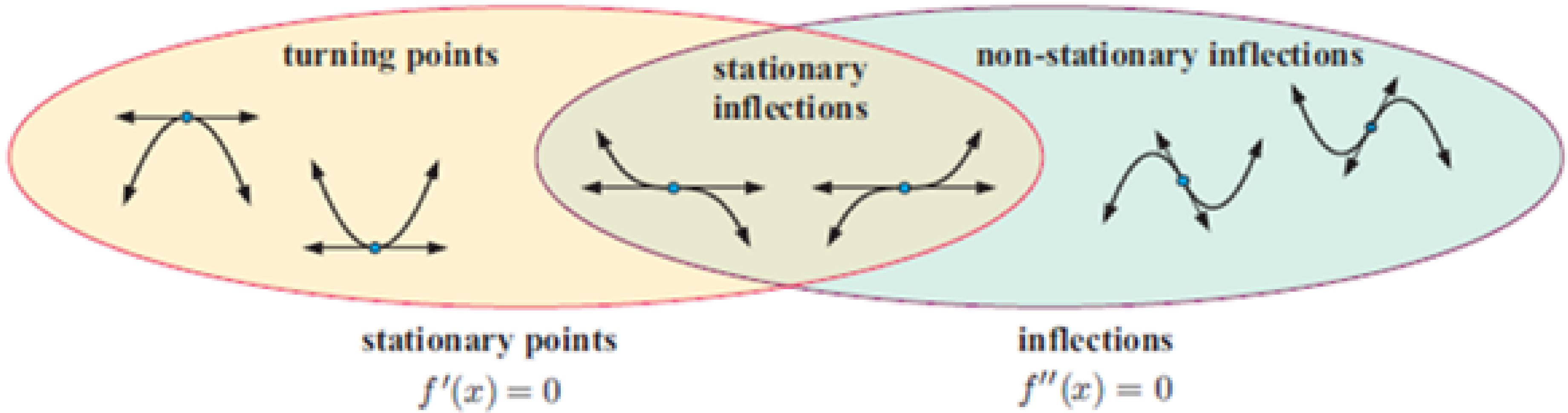
Sign diagram for $f''(x)$

Slopes are decreas

$-$	b	$+$

Slopes are increasing

SUMMARY


turning points
stationary inflections
non-stationary inflections
stationary points
$f'(x) = 0$
inflections
$f''(x) = 0$

Example: Find and classify all points of inflection (pg 404 #2c)

A) $f(x) = x^3 - 6x^2 + 9x + 1$

Stationary $f'(x) = 0$

$f'(x) = 3x^2 - 12x + 9$

$0 = 3x^2 - 12x + 9$

$0 = x^2 - 4x + 3$

$0 = (x-3)(x-1)$

$x = 3 \quad x = 1$

$+ \mid - \mid +$

$0 \quad 1 \quad 3$

$x = 1$ local max

$x = 3$ local min

$\therefore$ there are no stationary inflection pts

nonstationary $f''(x) = 0$

$f''(x) = 6x - 12$

$0 = 6x - 12$

$12 = 6x$

$2 = x$

$- \mid +$

$0 \quad 2$

CD CU

Example: pg 404 #3g

3 For each of the following functions:

- **i** Find and classify all points where $f'(x) = 0$
- **ii** Find and classify all points of inflection
- **iii** Find intervals where the function is increasing or decreasing
- **iv** Find intervals where the function is concave up or down
- **v** Sketch the function showing the features you have found.

$f(x) = x^4 - 4x^2 + 3$

i) $f'(x) = 4x^3 - 8x$

$0 = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$= 4x(x - \sqrt{2})(x + \sqrt{2})$

$x = 0 \quad x = -\sqrt{2}$

$x = \sqrt{2}$

$-$ | $+$ | $-$ | $+$

$-\sqrt{2} \quad 0 \quad \sqrt{2}$

fill into $f(x)$

$x = -\sqrt{2}$ local min $(y = -1)$

$x = 0$ local max $(y = 3)$

$x = \sqrt{2}$ local min $(y = -1)$

ii) $f''(x) = 0$

$f''(x) = 12x^2 - 8$

$0 = 12x^2 - 8$

$0 = 4(3x^2 - 2)$

$0 = 3x^2 - 2$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

CU $+$ | CD | $+$ CU

$-\sqrt{\frac{2}{3}} \quad 0 \quad \sqrt{\frac{2}{3}}$

iii) inc/dec .... looking for

$f'(x)>0$ or $f'(x)<0$

Inc: $x\in(-\sqrt{2},0)\cup(\sqrt{2},\infty)$

dec: $x\in(-\infty,-\sqrt{2})\cup(0,\sqrt{2})$

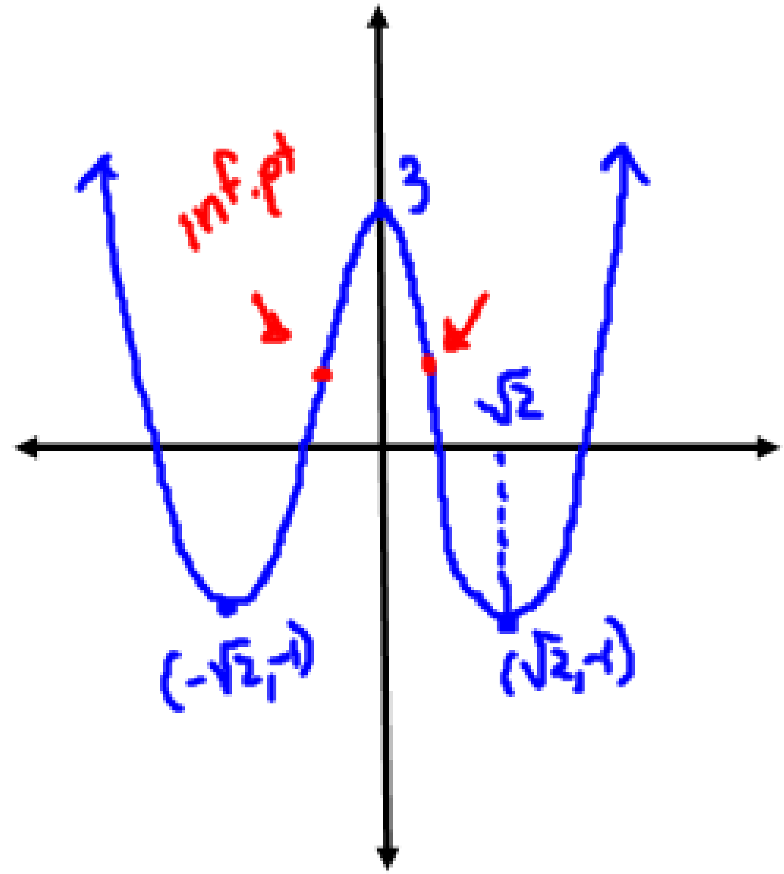
iv) CU (+ on signdiagram of $f''(x)$)

$x\in\left(-\infty,-\sqrt{\tfrac{2}{3}}\right)\cup\left(\sqrt{\tfrac{2}{3}},\infty\right)$

CD: (− on sign diagram of $f''(x)$)

$x\in\left(-\sqrt{\tfrac{2}{3}},\sqrt{\tfrac{2}{3}}\right)$

e) Sketch



x-int: $f(x)=0$

$0=x^4-4x^2+3$

$=(x^2-3)(x^2-1)$

$=(x-\sqrt{3})(x+\sqrt{3})(x-1)(x+1)$

$x=-\sqrt{3},\ x=\sqrt{3},\ x=1,\ x=-1$