The quotient of functions:

$$h(x) = \frac{f(x)}{g(x)} \qquad h(x) = \left(\frac{f}{g}\right)(x)$$

The domain of a quotient of functions is restricted for values of x where $g(x) = 0$.

$$h(x) = \frac{x+2}{x-3} \leftarrow \text{VA}$$

GY

$$h(x) = \frac{x^2+5x+6}{x+3} = \frac{(x+3)\cancel{(x+2)}}{\cancel{x+2}} \leftarrow \text{POD at } x=-2$$

Pg. 493

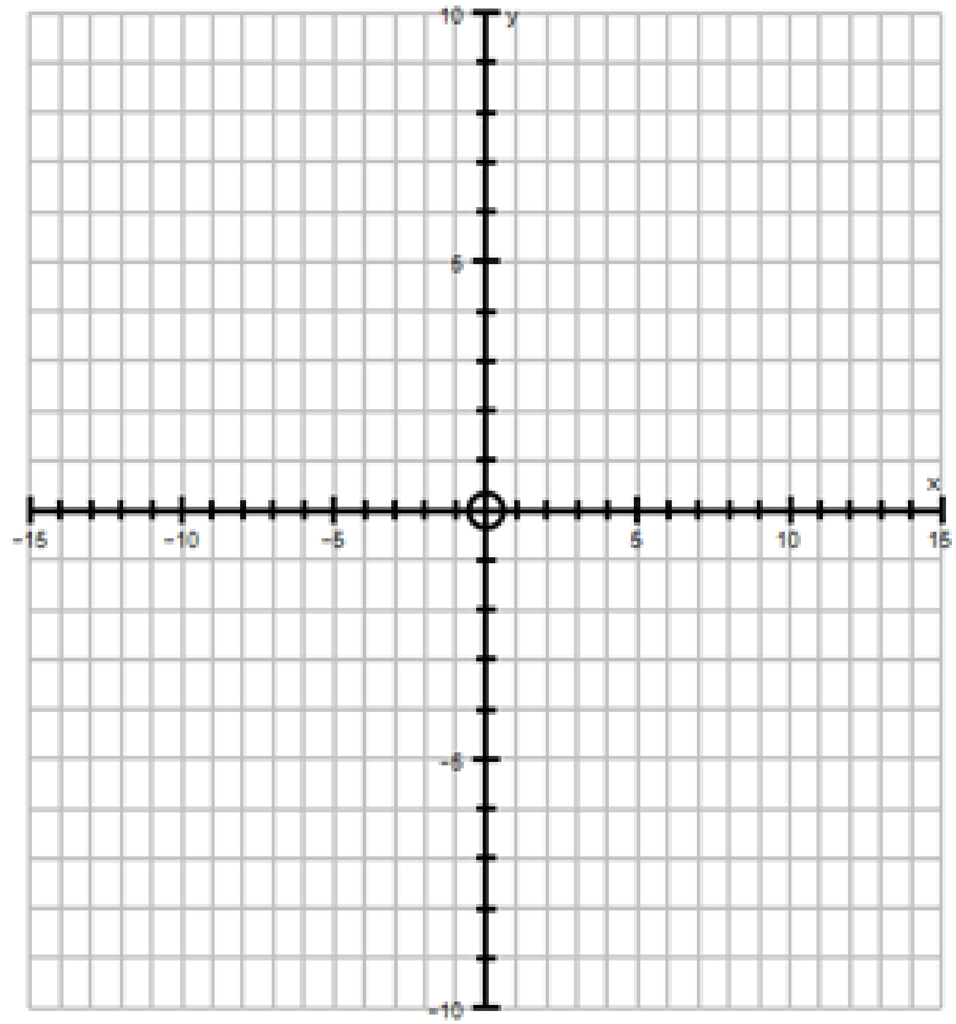
Your Turn

Let $f(x) = x + 2$ and $g(x) = x^2 + 9x + 14$.

a) Determine the equation of the function $h(x) = \left(\frac{f}{g}\right)(x)$.

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain and range of $h(x)$.



10.3

Composite Functions

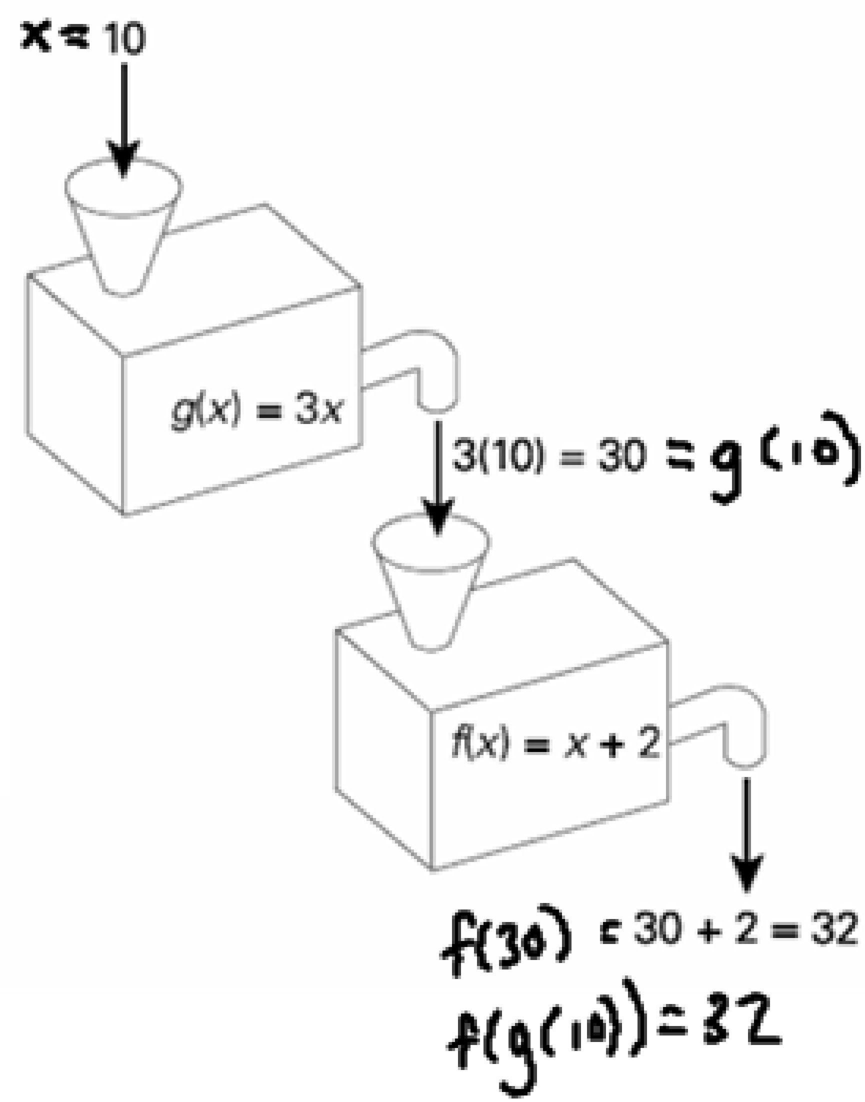
Focus on...

- determining values of a composite function
- writing the equation of a composite function and explaining any restrictions
- sketching the graph of a composite function

Composite functions are functions that are formed from two functions in which the result of one of the functions is used as the input for the other function.

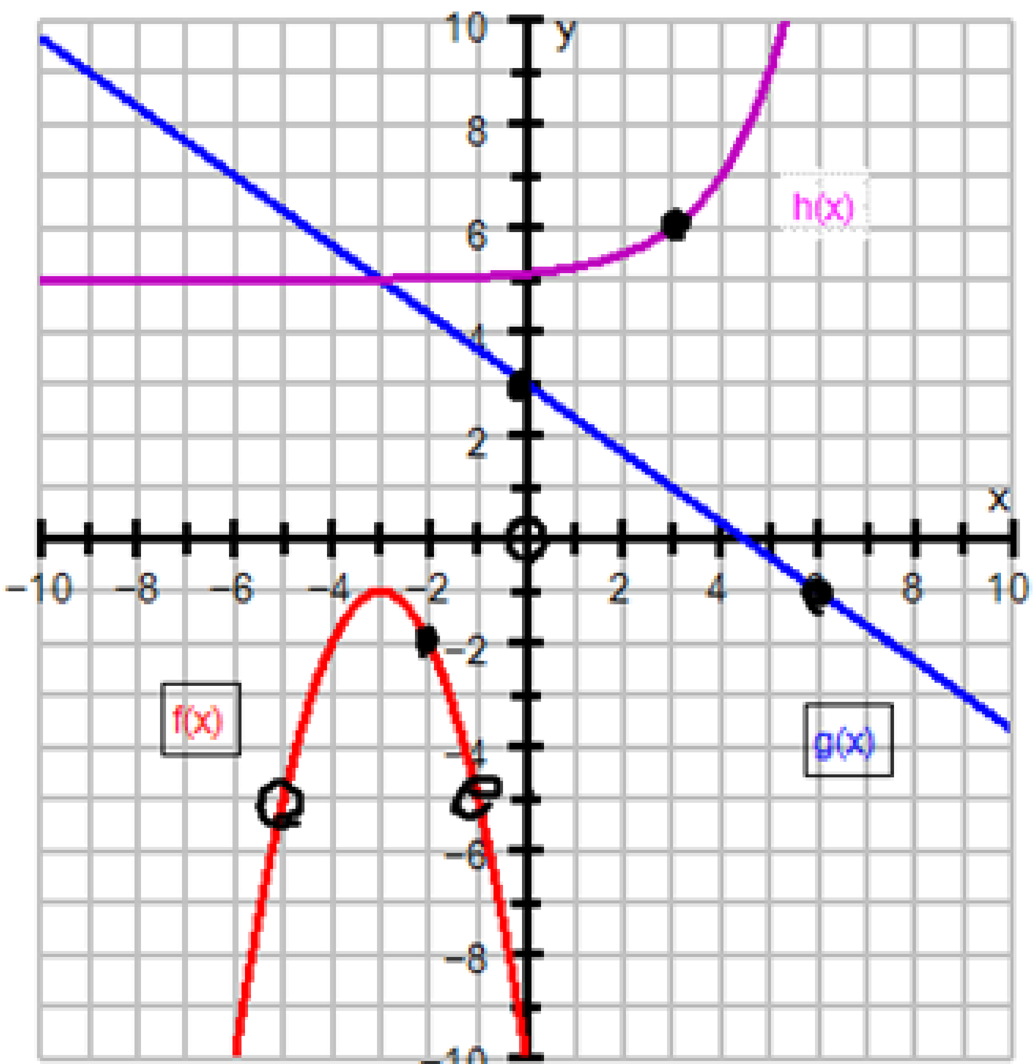
Given $f(x)$ and $g(x)$, the **composite function** of f and g will convert x into $f(g(x))$.

f of g of x, or f at g of x



Examples: $y = f(x)$

1.



Find:

(a) $\overset{x}{f(-2)} = -2$

(b) x if $f(x) = -5$

$x = -5$ and $x = -1$

(c) $g(h(3)) = g(6) = -1$

$h(3) = 6$ $\quad g(h(3)) = -1$

(d) $h(g(0)) = h(3)$

$g(0) = 3$ $\quad = 6$

2. If $f(x) = x^2$ and $g(x) = x - 3$, find:

(a) $f(g(x)) = f(x-3)$

$= (x-3)^2$

$= (x-3)(x-3)$

$= x^2 - 6x + 9$

(b) $g(f(x)) = f(x) - 3$

$= (x^2) - 3$

$= x^2 - 3$

$$f(g(x)) \neq g(f(x))$$

Say "g" of "f" of "x"

$f(x)$ $g(x)$ $h(x)$ $p(x)$ $q(x)$ $j(x)$

$$f(g(h(p(q(j(f(x)))))))$$

$$(f \circ g \circ h \circ p \circ q \circ j \circ f)(x)$$

open circle

multiply $y = f(x) \cdot g(x)$
$= (f \cdot g)(x)$

composite $y = f(g(x))$
$= (f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$

3. If $f(x) = 2x + 3$ and $g(x) = \sqrt{x-1}$, determine each of the following.

(a) $(g \circ f)(x)$

$$g(f(x)) = g(2x+3)$$
$$= \sqrt{(2x+3)-1}$$
$$= \sqrt{2x+2}$$

(b) $f(g(5))$

$$g(5) = \sqrt{5-1}$$
$$= \sqrt{4}$$
$$= 2$$

$$f(g(5)) = f(2)$$
$$= 2(2)+3$$
$$= 7$$

4. Given: $g(x) = -5x - 3$ and $j(x) = 2x + 7$. Find x if $(g \circ j)(x) = -29$

$$g(j(x)) = -29$$

$$g(2x+7) = -29$$

$$-5(2x+7) - 3 = -29$$

$$-10x - 35 - 3 = -29$$

$$-10x - 38 = -29$$

$$-10x = -29 + 38$$

$$-10x = 9$$

$$x = \frac{-9}{10}$$

Pg. 503

Your Turn

Given the functions $f(x) = \sqrt{x - 1}$ and $g(x) = -x^2$, determine $(g \circ f)(x)$. Then, state the domain of $f(x)$, $g(x)$, and $(g \circ f)(x)$.

$$g(f(x)) = g(\sqrt{x-1})$$
$$= -(\sqrt{x-1})^2$$
$$= -(x-1)$$
$$= -x+1$$

linear

$f(x) = \sqrt{x-1}$ ← radical

D: $\{x \mid x \geq 1\}$

$g(x) = -x^2$ ← quadratic

D: $\{x \in R\}$

$g(f(x)) = -x+1$

D: $\{x \mid x \geq 1\}$

Example:

If $f(x) = x + 5$ and $g(f(x)) = x^2 + 10x + 18$, find $g(x)$.

$g(f(x)) = x^2 + 10x + 18 \leftarrow$ put in vertex form

Vertex: $y = a(x-h)^2 + k$

factored: $y = a(x-\alpha)(x-\beta)$

$\leftarrow$ complete the square

$= (x^2 + 10x) + 18$

$= (x^2 + 10x + 25) + 18 - 25$

$= (x+5)^2 - 7$

$g(f(x)) = (f(x))^2 - 7$

$g(x) = (x)^2 - 7$

Pg. 505

Your Turn

If $h(x) = f(g(x))$, determine $f(x)$ and $g(x)$.

$$h(x) = \sqrt[3]{x} + \frac{3}{3 + \sqrt[3]{x}}$$

HW pg 507
#1-7

$$f(g(x)) = \sqrt[3]{x} + \frac{3}{3+\sqrt[3]{x}}$$

→ look for repeated term(s) in composite function

$$\sqrt[3]{x}$$

→ assign the repeated term(s) as a function; $g(x) = \sqrt[3]{x}$

$$f(g(x)) = g(x) + \frac{3}{3+g(x)}$$

→ replace $g(x)$ with x to find outer function

$$f(x) = x + \frac{3}{3+x}$$

HW: pg 507 #1-7