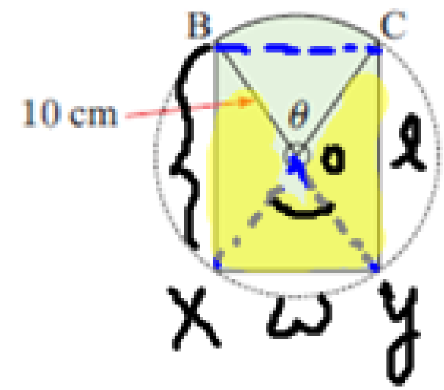
Pg 433

12 A circular piece of tinplate of radius 10 cm has 3 segments removed as illustrated. The angle θ is measured in radians.

a Show that the remaining area is given by $A = 50(\theta + 3\sin\theta)$ cm^2.

b Find θ such that the area A is a maximum, and also the area A in this case.



$$A_{SHADED} = A_{sector} + \frac{3}{4}A_{\square}$$

$$= \frac{1}{2}\theta r^2 + \frac{3}{4}(\ell \times \omega)$$

Find BC using cosine formula

$$BC^2 = BO^2 + CO^2 - 2(BO)(CO)\cos\theta$$

$$BC^2 = 10^2 + 10^2 - 2(10)(10)\cos\theta$$

$$BC^2 = 200 - 200\cos\theta$$

$$BC = \sqrt{200 - 200\cos\theta}$$

Use Pythag. Thm to find BX

$$a^2+b^2=c^2$$

$$Bx^2+Bc^2=20^2$$

$$Bx^2=400-(200-200\cos\theta)$$

$$Bx^2=200+200\cos\theta$$

$$Bx=\sqrt{200+200\cos\theta}$$

$$A_{SHADED}=A_{sector}+\frac{3}{4}A_{\square}$$

$$=\frac{1}{2}\theta r^2+\frac{3}{4}(l\times w)$$

$$=\frac{1}{2}\theta r^2+\frac{3}{4}\left(\sqrt{200+200\cos\theta}\right)\left(\sqrt{200-200\cos\theta}\right)$$

$$=\frac{1}{2}\theta r^2+\frac{3}{4}\left(\sqrt{40000-40000\cos^2\theta}\right)$$

$$\sqrt{a-ab}\sqrt{a+ab}$$

$$\sqrt{(a-ab)(a+ab)}$$

$$\sqrt{a^2-a^2b^2}$$

$$\sqrt{a^2(1-b^2)}$$

$$a\sqrt{1-b^2}$$

$$A_{SA} = \frac{1}{2}\theta r^2 + \frac{3}{4}\left(200\sqrt{1-\cos^2\theta}\right)$$ trig Identity

$\sin^2\theta = 1-\cos^2\theta$

$$= \frac{1}{2}\theta(10)^2 + \frac{3}{4}\left(200\sqrt{\sin^2\theta}\right)$$

$$= 50\theta + \frac{3}{4}(200\sin\theta)$$

$$= 50\theta + 150\sin\theta$$

A) $A = 50(\theta + 3\sin\theta)$

B) $\dfrac{dA}{d\theta} = 50 + 150(\cos\theta)$

$$\frac{dA}{d\theta} = 50 + 150\cos\theta$$

$0 = 50 + 150\cos\theta$

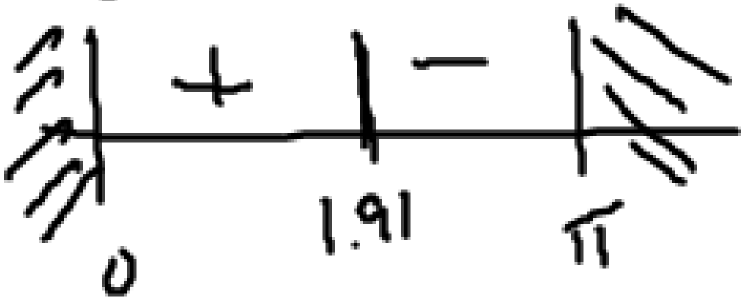
$\cos\theta = -\frac{1}{3}$

$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

$\theta = 1.91$ radians

$0 < \theta < \pi$

sign diagram for A'



if $\theta = 0$, $A = 0$

$\theta = \pi$, $A = 0$

$\theta = 1.91$, $A = 50(1.91 + 3\sin(1.91))$

$A = 236.95$